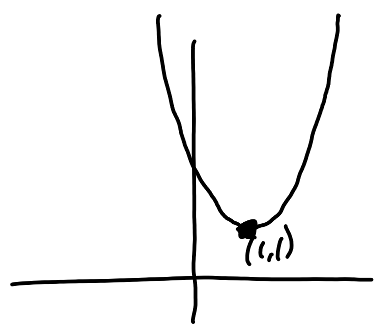
Section 3.5 Complex numbers as roots of polynomials.

(aka "zero")

Again, a root, r of a polynomial $P(x)$ is number r for which $P(r)=0$.

If we only allow r to be a real number, Then we are left with polynomials with no roots.

example



$y=(x-1)^2+1$

$(x-1)^2+1=0$

has no solutions because The solutions must be x-intercepts for The graph.

So we expand our notions of what numbers are and see if That helps.

Def let $i=\sqrt{-1}$.

Therefore $i^2=-1$, $i^3=-1\cdot i=-i$, $i^4=(i^2)^2=1$

The number i is called "imaginary"

A <u>complex number</u> is of the form

$$a+bi$$

where a and b are real numbers.

Now we can do arithmetic with complex numbers

<u>addition</u>

$$2-3i+7i = 2+4i$$

$$2+3i-(1-7i) = 1+10i$$

$$\boxed{i^2=-1}$$

<u>multiplication</u>

$$(2-3i)\,i = 2i-3i^2 = 2i-3(-1)$$
$$= -3+2i$$

Now every quadratic polynomial either has 2 roots

or one root of multiplicity 2.

<u>example</u>

$$x^2-10x+25 = 0$$

$$(x-5)^2 = 0$$

$x=5$ is a root of multiplicity 2.

example $(x-1)^2+1=0$

$$(x-1)^2=-1$$

$$x-1=\pm\sqrt{-1}$$

$$x-1=\pm i$$

$$x=1\pm i$$

$x=1+i$ and $x=1-i$

are the roots of $P(x)=(x-1)^2+1=x^2-2x+2$

example what are the roots of x^2-x+1

quadratic formula

$$x=\frac{1\pm\sqrt{1-4}}{2}=\frac{1\pm\sqrt{-3}}{2}=\frac{1\pm\sqrt{3(-1)}}{2}$$

$$=\frac{1\pm\sqrt{3}\sqrt{-1}}{2}=\frac{1\pm\sqrt{3}\,i}{2}$$

$x=\frac{1+\sqrt{3}i}{2}$ and $x=\frac{1-\sqrt{3}i}{2}$

$x=\frac{1}{2}+\frac{\sqrt{3}}{2}i$ $x=\frac{1}{2}-\frac{\sqrt{3}}{2}i$

Fundamental Theorem of Algebra

Every polynomial of degree n has n roots counting multiplicities that are all real and/or complex.

<u>example</u> Find all of the roots of x^4+5x^2+4

$x^4+5x^2+4=0$

$(x^2)^2+5(x^2)+4=0$

Think it
$a^2+5a+4=0$ with $a=x^2$
$(a+4)(a+1)=0$

$(x^2+4)(x^2+1)=0$

$x^2+4=0$ or $x^2+1=0$

$x^2=-4$ $\qquad$ $x^2=-1$

$x=\pm\sqrt{-4}$ $\qquad$ $x=\pm\sqrt{-1}$

$x=\pm\sqrt{4(-1)}$ $\qquad$ $x=\pm i$

$x=\pm\sqrt{4}\sqrt{-1}$

$x=\pm 2i$

So the four roots of x^4+5x^2+4 are

$i, -i, 2i, -2i$