Section 2.7 Ways of Combining functions.

Given two functions $f(x)$ and $g(x)$ with common domains of real numbers, These functions can be combined algebraically as follows.

Definitions

$$(f+g)(x) = f(x) + g(x)$$

$$(f-g)(x) = f(x) - g(x)$$

$$(fg)(x) = f(x)\,g(x)$$

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$$

example

Let $f(x) = \frac{2}{x+1}$ $g(x) = \frac{x}{x+1}$.

Then

$$(f+g)(x) = \frac{2}{x+1} + \frac{x}{x+1} = \frac{2+x}{x+1}$$

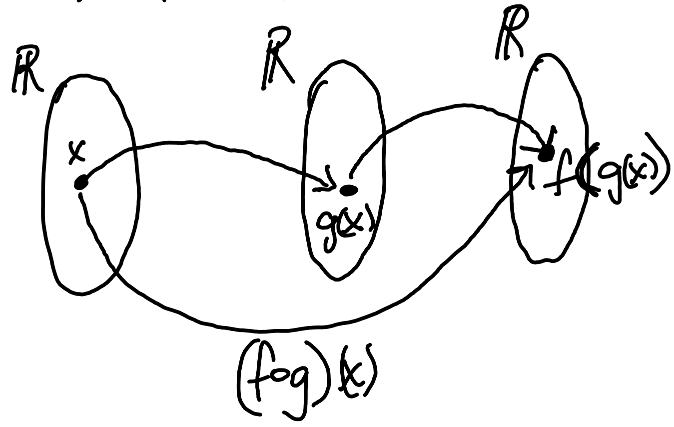
$$(f-g)(x) = \frac{2}{x+1} - \frac{x}{x+1} = \frac{2-x}{x+1}$$

$$(fg)(x) = \frac{2}{x+1}\,\frac{x}{x+1} = \frac{2x}{(x+1)^2}$$

$$\left(\frac{f}{g}\right)(x) = \frac{\frac{2}{x+1}}{\frac{x}{x+1}} = \frac{2}{\cancel{x+1}}\,\frac{\cancel{x+1}}{x} = \frac{2}{x}$$

Composition of functions

If $f(x)$ and $g(x)$ are function on the real numbers Then consider The following diagram.



This function said "eff following gee" is called a composition of functions.

$$(f \circ g)(x) = f(g(x))$$

example $\quad f(x) = \frac{2}{2x-1} \qquad g(x) = \frac{x}{x+1}$

$$(f \circ g)(x) = f(g(x)) = f\left(\frac{x}{x+1}\right) = \frac{2}{2\left(\frac{x}{x+1}\right) - 1} = \underset{\substack{\uparrow \\ \text{simplify}}}{\cdots} = \frac{2x+2}{x-1}$$

$$(g \circ f)(x) = g(f(x)) = g\left(\frac{2}{2x-1}\right) = \frac{\frac{2}{2x-1}}{\frac{2}{2x-1}+1} = \underset{\substack{\uparrow \\ \text{simplify}}}{- - - -} = \frac{2}{2x+1}$$

So $f \circ g \neq g \circ f$ in general.