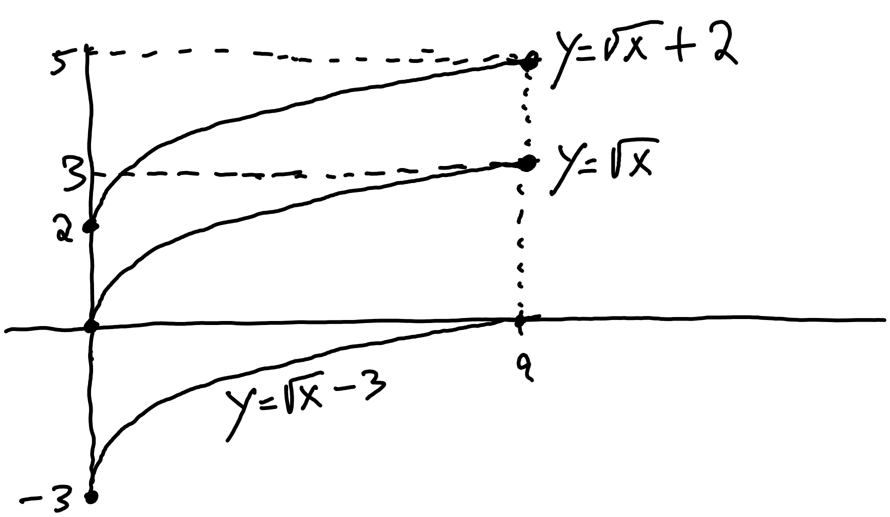
Section 2.6 Transformations of Functions

Given the graph of a function $y = f(x)$ there are several standard transformations to the function which affect the graph in predictable ways.

Vertical Shifting

Given $y = f(x) + C$ it's graph is obtained from
$y = f(x)$ by shifting up C units when $C > 0$
by shifting down C units when $C < 0$.

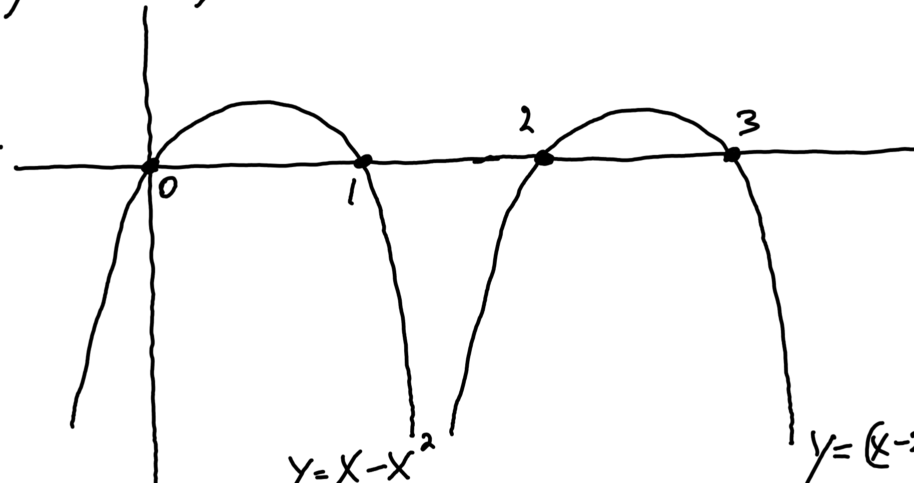
example



Horizontal Shifting

Let $c>0$. $y=f(x-c)$ is obtained from $y=f(x)$ by shifting to the right by c units. $y=f(x+c)$ is obtained from $y=f(x)$ by shifting to the left c units.

<u>example</u>



$y=x-x^2$

$f(x)=x-x^2$

$y=(x-2)-(x-2)^2$

$=x-2-(x^2-4x+4)$

$=-x^2+5x-11$

$f(x-2)=(x-2)-(x-2)^2$

<u>example</u> combining both horizontal and vertical shifting.

shifted left 1 up 2.

$y=f(x)$ goes to $y=f(x+1)+2$

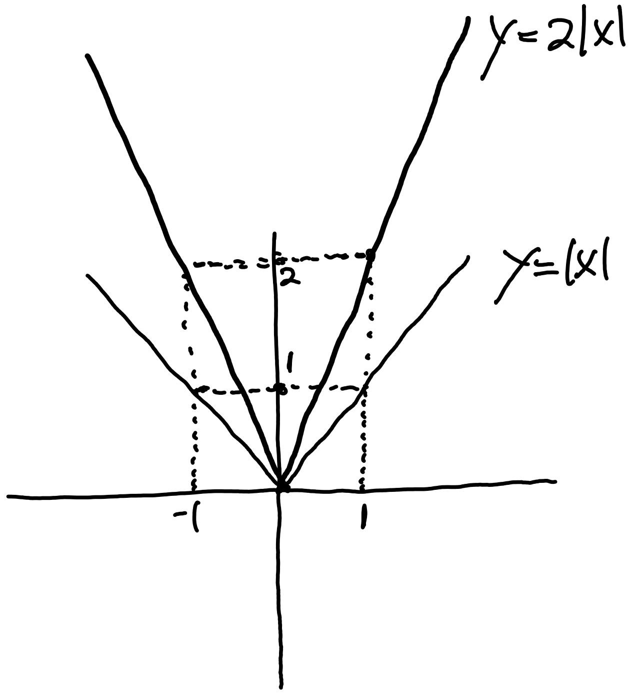
Vertical stretching (compressing)

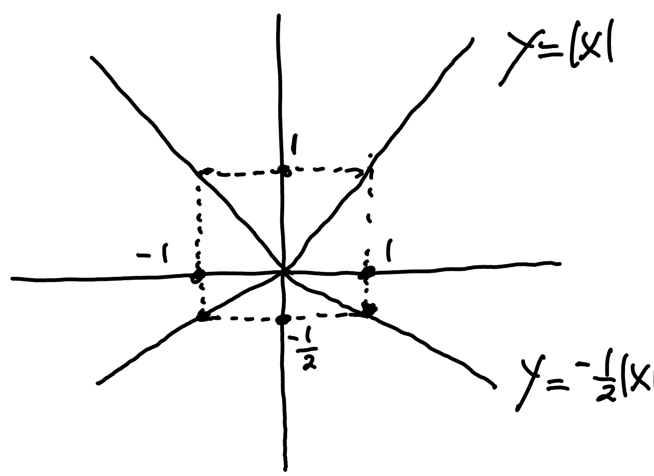
If $k>1$, then $y=kf(x)$ is obtained from $y=f(x)$ by stretching upward by a factor of k.

If $0<k<1$, then $y=kf(x)$ is obtained from $y=f(x)$ by compressing downward by a factor of k.

If $k<0$, then the stretch/compress is accompanied by a reflection through the x-axis.

<u>example</u> $y=|x|=\begin{cases} x & \text{if } x\geq 0 \\ -x & \text{if } x<0 \end{cases}$





Stretch/compress horizontally

Given $y=f(x)$, *

if $k>1$,

Then $y=f(kx)$ is obtained by compressing $y=f(x)$ a factor of $\frac{1}{k}$.

and $y=f(\frac{1}{k}x)$ is obtained by stretching $y=f(x)$ a factor of k.

if $k<0$, Then The stretching/compressing is accompanied by a reflection through The y-axis.

example