2.4 Average rate of change.

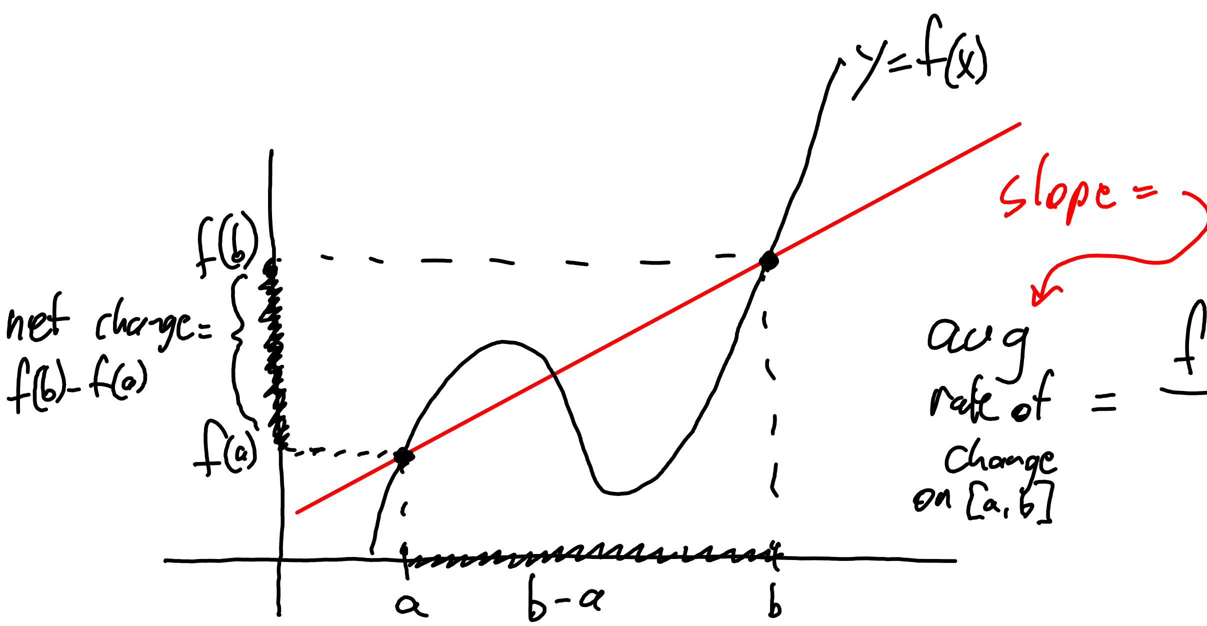
Given a function $f(x)$ on a closed interval $[a,b]$, $a \le x \le b$

The **net change** of $f(x)$ on $[a,b]$ is

$$\text{net change} = f(b) - f(a)$$

The **average rate of change** on $[a,b]$ is

$$\text{avg rate of change } [a,b] = \frac{f(b) - f(a)}{b - a}$$



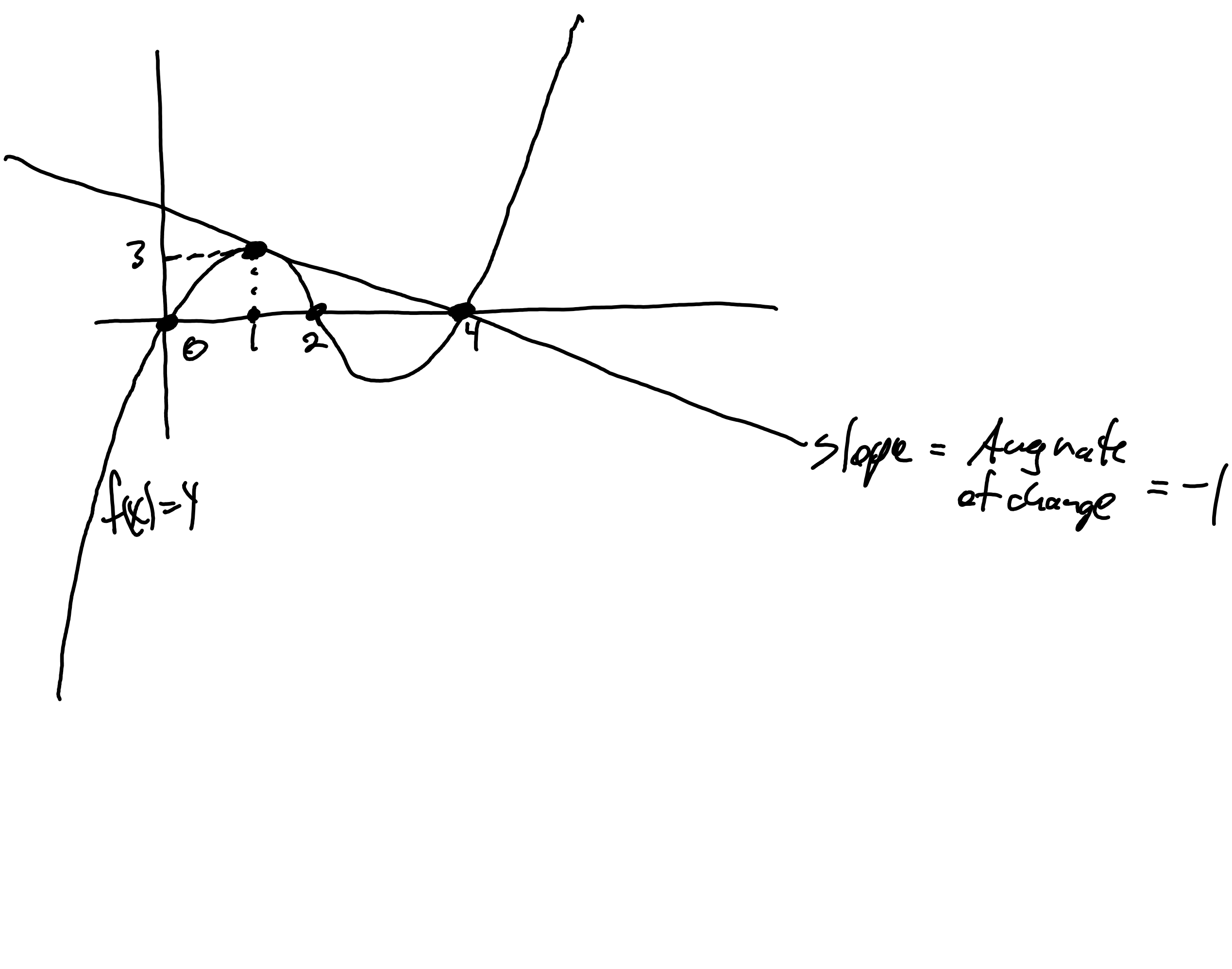
$$\text{avg rate of change on } [a,b] = \frac{f(b) - f(a)}{b - a}$$

example $f(x) = x(x-2)(x-4)$

Find the net change and avg rate of change for $f(x)$ on $1 \le x \le 4$.

Net change $= f(4) - f(1) = 4(2)(0) - (1)(-1)(-3) = 0 - 3 = -3$

$$\text{Avg rate of change} = \frac{f(4) - f(1)}{4-1} = \frac{-3}{3} = -1$$



Rates of change and units

If x is measured in units P

and

$y = f(x)$ is measured in units Q,

Then $\underset{a \leq x \leq b}{\text{avg rate of change}} = \dfrac{f(b)-f(a)}{b-a}$ which is in $\dfrac{\text{units } Q}{\text{units } P}$

or "units of Q per P".

Central example

If $f(t)$ is measured in some units of distance

(cm, inches, feet, km, miles, ..etc.)

and t is measured in some units of time

(seconds, minutes, hrs, ...etc..)

Then

$\underset{a \leq t \leq b}{\text{Avg rate of change}} = \dfrac{f(b)-f(a)}{b-a}$ is in $\dfrac{\text{distance}}{\text{time}}$ which is a unit of "velocity" example Let $f(t) = -t^2 + 2t + 1$ feet at t seconds be the height of a ball thrown vertically upwards @ $t = 0$ seconds.

Find the average velocity (i.e., average rate of change) for the ball on $0 \le t \le 1$ and $1 \le t \le 2$.

$$\underset{0 \le t \le 1}{\text{Avg velocity}} = \frac{f(1) - f(0)}{1 - 0} = \frac{(-1+2+1) - (0+0+1)}{1-0} = \frac{2-1}{1-0} = 2 \ \frac{\text{feet}}{\text{sec}}$$

$$\underset{1 \le t \le 2}{\text{avg velocity}} = \frac{f(2) - f(1)}{2 - 1} = \frac{(-4+4+1) - (-1+2+1)}{2-1} = \frac{1-2}{1} = -1 \ \frac{\text{feet}}{\text{sec}}$$

A positive average velocity indicates the ball is mostly moving upward for $0 \le t \le 1$

A negative average velocity indicates the ball is mostly moving downward for $1 \le t \le 2$.