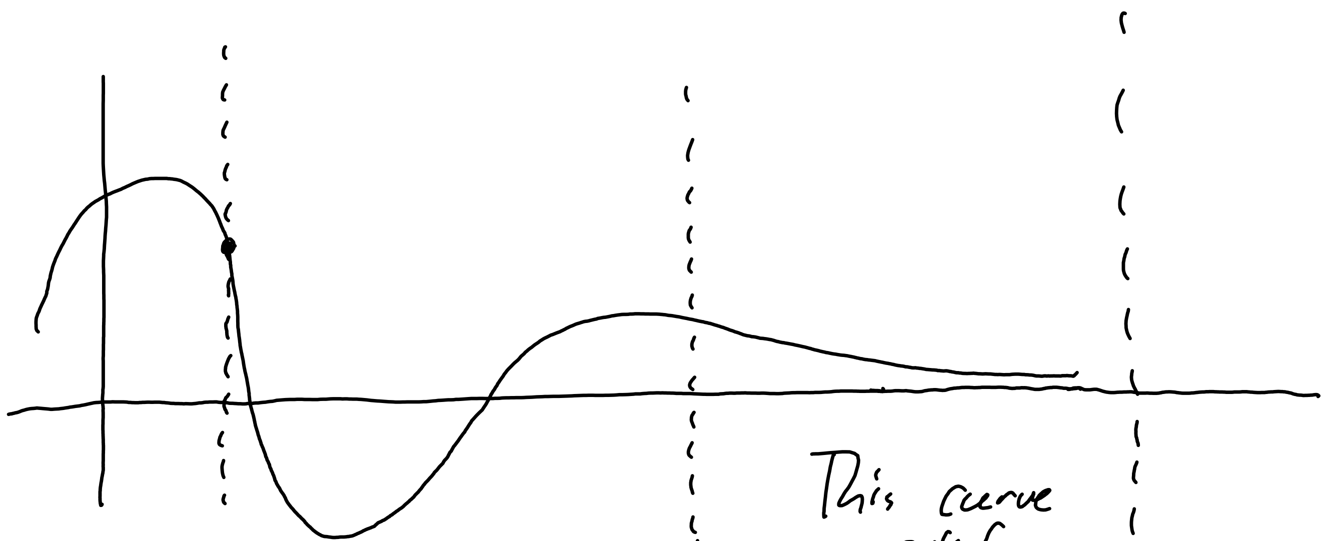
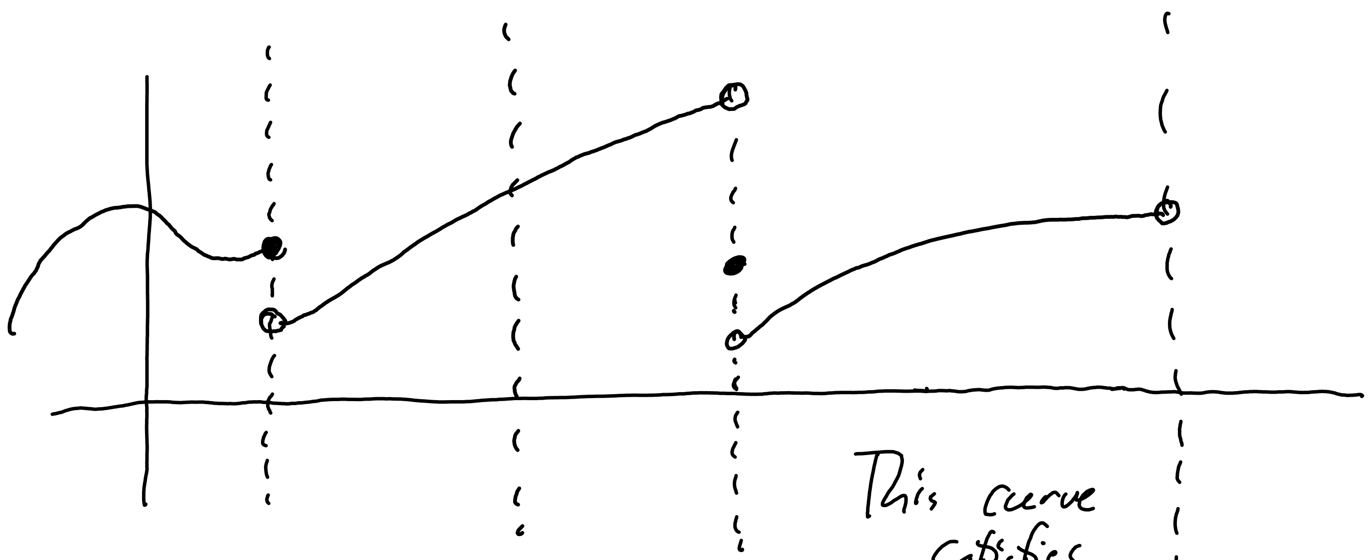
Section 2.2

Given a curve in the xy-plane, we say that it passes the "vertical line test" when every vertical line in the plane intersects the curve at most once.

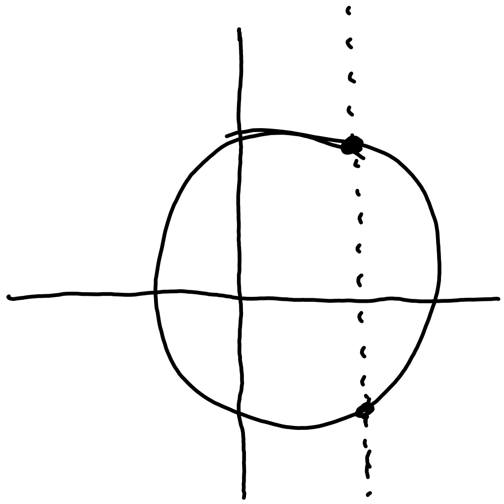


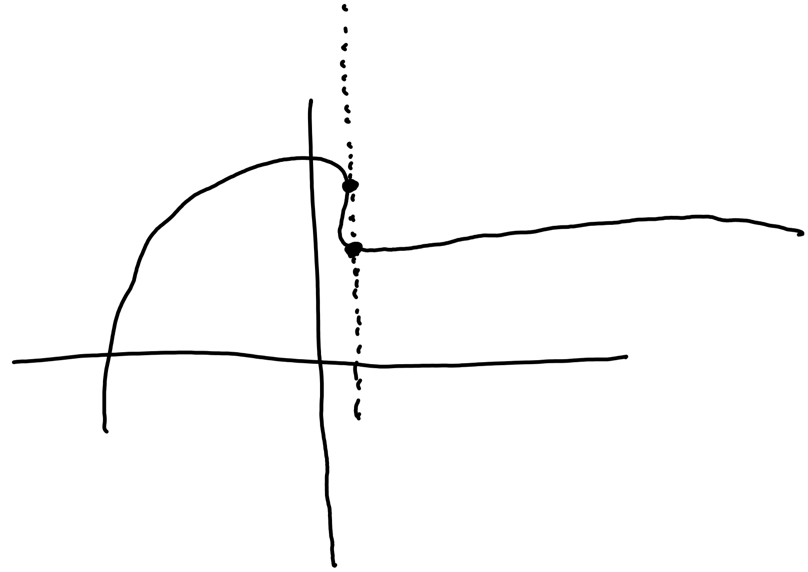
This curve satisfies the vertical line test.



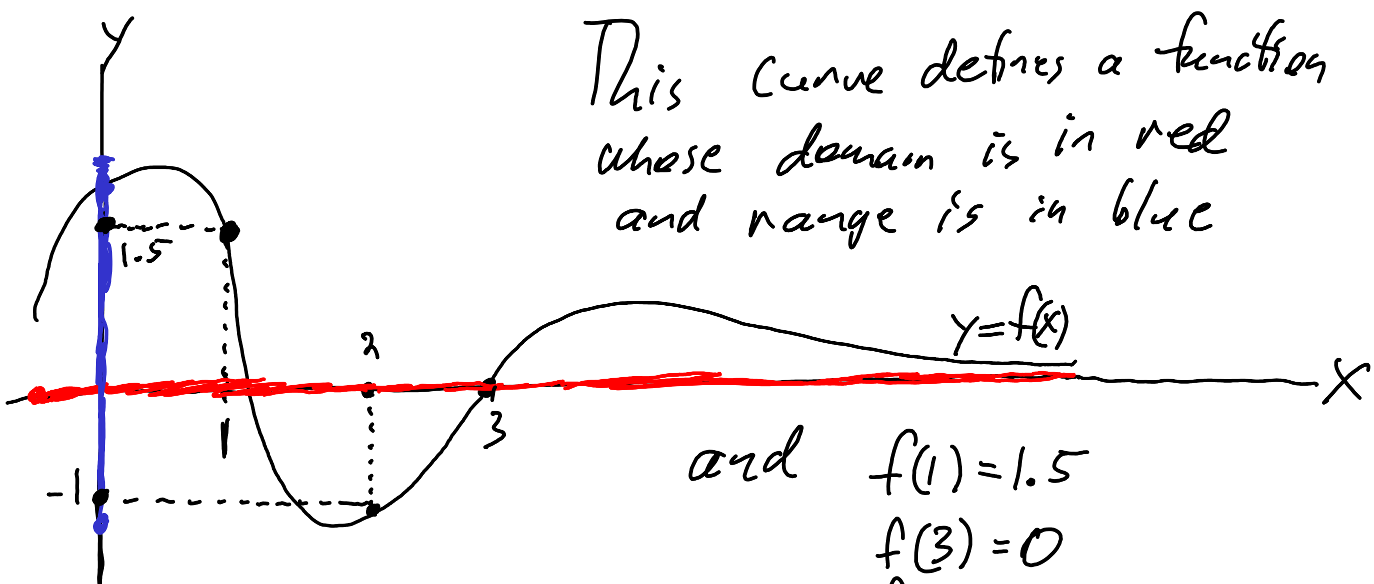
This curve satisfies the vertical line test.

These curves don't pass the vertical line test





A curve which passes the vertical line test gives a function where domain values are on the x-axis and range values are on the y-axis with the correspondence determined by the curve.



This curve defines a function whose domain is in red and range is in blue

and $f(1)=1.5$

$f(3)=0$

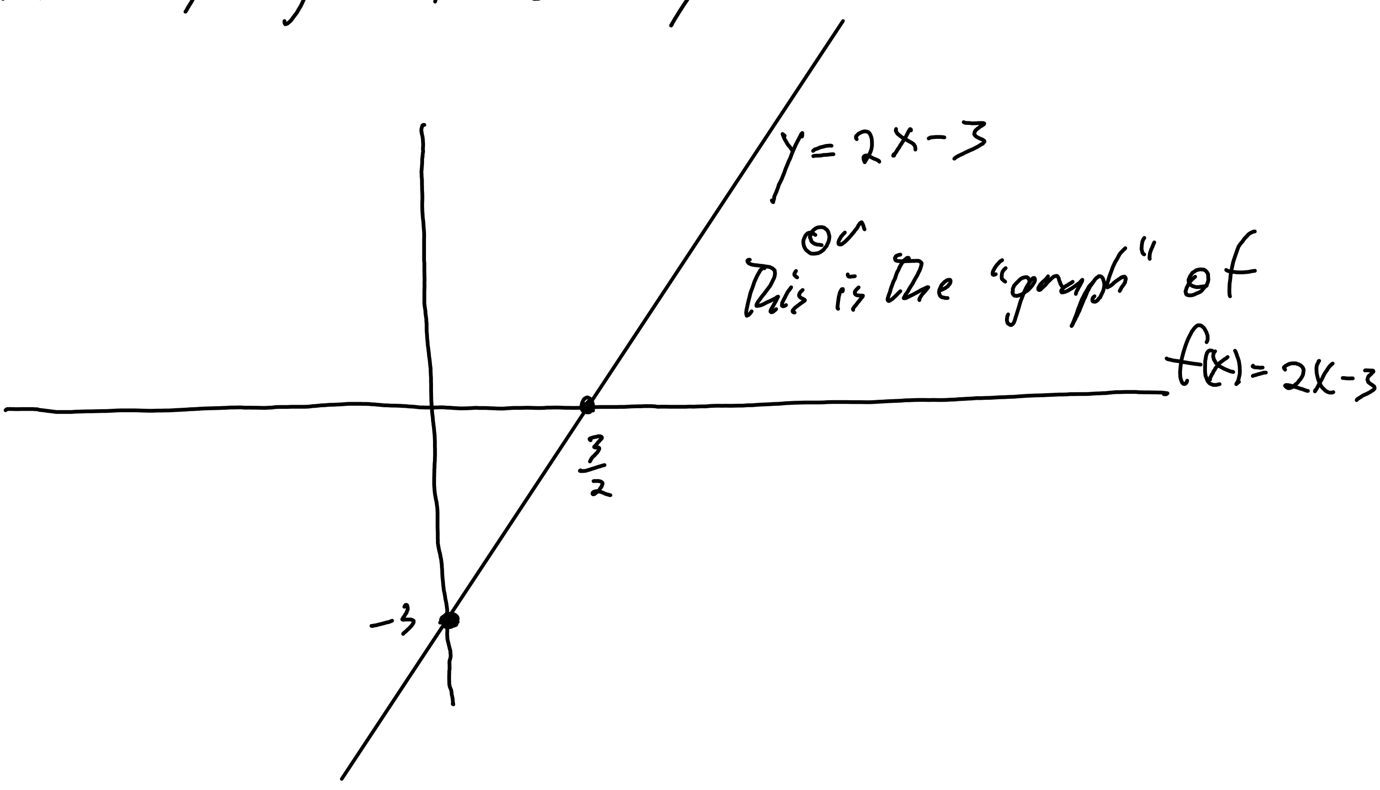
$f(2)=-1$

The function corresponding to this curve is denoted by $y = f(x)$.

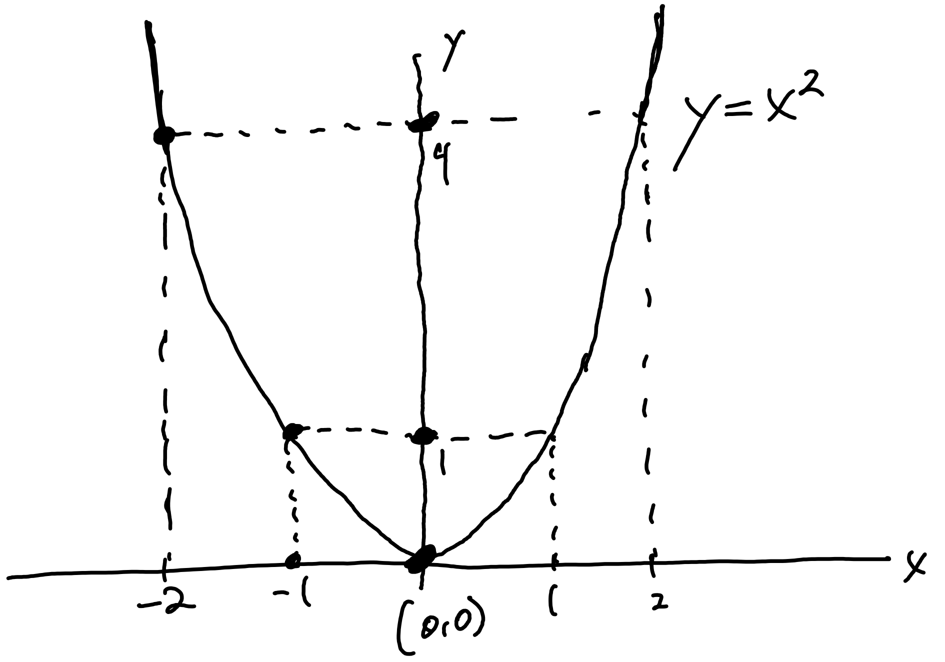
Conversely, if a function $f(x)$ has a domain which is an interval, then it defines a curve which passes the vertical line test.

<u>example</u>

If $\boxed{f(x) = 2x-3}$, then its "<u>graph</u>" is just denoted by $y=f(x)$ or $y = 2x-3$



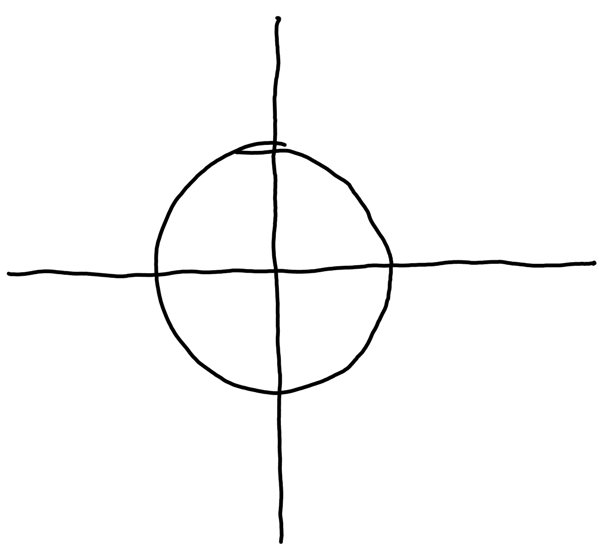
example The function $f(x)=x^2$ has the following graph.



example The equation of the circle $x^2+y^2=1$

defines the entire circle of radius $=1$

center $=(0,0)$



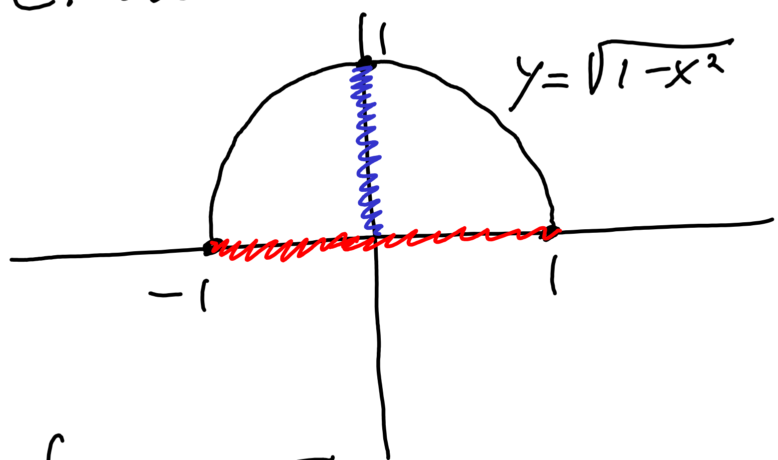
doesn't pass vertical line test.

But $x^2+y^2=1$

$y^2=1-x^2$

$y=\pm\sqrt{1-x^2}$ still doesn't give a function

But $y = \sqrt{1-x^2}$ defines the top half of the circle



So $f(x) = \sqrt{1-x^2}$

is a function with domain $[-1,1]$

range $[0,1]$

Example

$$f(x) = \begin{cases} \frac{1}{2}x + 1 & \text{if } x < -4 \\ 2x + 3 & \text{if } -4 \le x < 1 \\ -x & \text{if } x \ge 1 \end{cases}$$

$f(-4) = 2(-4) + 3 = -5$