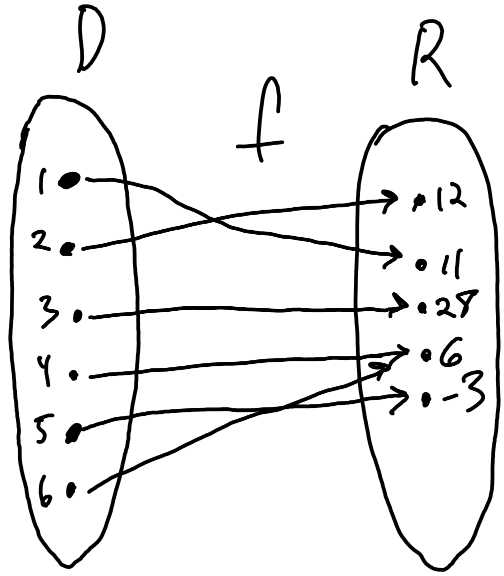
Section 2.1 Functions

"eff of x"

A function written as $f(x)$ has input value (or argument) x and output value $f(x)$. The input values come from some "Domain" and output values from some "Range."



This diagram is describing the function $f(x)$ for which

$f(1) = 11$, $f(2) = 12$

$f(3) = 28$, $f(4) = 6$,

$f(5) = -3$, $f(6) = 6$.

Note Every input value gets one output value, but one output value may be associated with more than one input values.

One of the most common ways of describing functions whose domains and ranges are numbers is with algebraic expressions.

example If we say that $f(x) = x^2+1$, then this means that

$$f(0) = 0^2+1 = 1$$
$$f(1) = 1^2+1 = 2$$
$$f(-1) = (-1)^2+1 = 2$$
$$\ddots \text{etc.}$$

Sometimes rather than just plugging in numbers to functions defined with algebraic expressions, we can plug in other algebraic expressions to get more, but related, functions.

example If $f(x) = \sqrt{x+3}$ then

$$f(1) = \sqrt{1+3} = \sqrt{4} = 2$$
$$f(-3) = \sqrt{-3+3} = \sqrt{0} = 0$$

$$f\left(\frac{t}{2}\right) = \sqrt{\frac{t}{2}+3}$$
$$f(t^2) = \sqrt{t^2+3}$$
$$f(x^2) = \sqrt{x^2+3}$$

$$\frac{f(x)}{2} = \frac{\sqrt{x+3}}{2}$$

$$\frac{f(t+1)-f(1)}{t} = \frac{\sqrt{t+1+3} - \sqrt{1+1}}{t}$$
$$= \frac{\sqrt{t+4} - \sqrt{2}}{t}$$

Example $f(x) = \dfrac{x}{x^2-1}$

$$f(4) = \frac{4}{4^2-1} = \frac{4}{16-1} = \frac{4}{15}$$

$$f(0) = \frac{0}{0^2-1} = \frac{0}{-1} = 0$$

$$f(1) = \frac{1}{1^2-1} = \frac{1}{0} \text{ undefined.}$$

In this case we say that 1 is not in the domain for $f(x)$.

$$f(t+2) = \frac{t+2}{(t+2)^2-1} = \frac{t+2}{t^2+4t+5}$$

$$f(t) + f(2) = \frac{t}{t^2-1} + \frac{2}{2^2-1}$$

$$= \frac{t}{t^2-1} + \frac{2}{3}$$

Notice $f(t+2) \neq f(t) + f(2)$

functional notation is not multiplication, even though it may look that way.

Domains

For an algebraically defined function, its domain is the largest set of real numbers on which the algebraic expression is defined.

<u>example</u> Find the domain for

$$f(x)=\frac{x^2}{x^2-x-6}$$

This algebraic expression is defined unless

$$x^2-x-6=0$$

$$(x+2)(x-3)=0$$

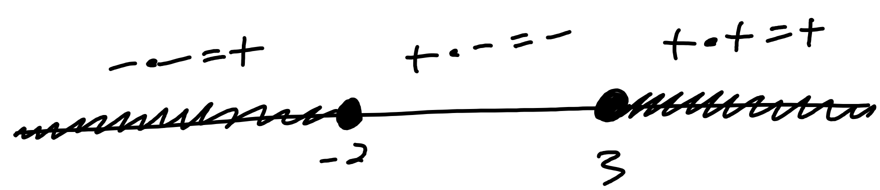
$$x=-2 \text{ or } x=3$$

Domain of $f(x)$ is all $x \neq -2$ or 3.

<u>example</u> $f(x)=\sqrt{x^2-x-6}$

Defined as long as $x^2-x-6 \geq 0$

$$(x-3)(x+2)\geq 0$$

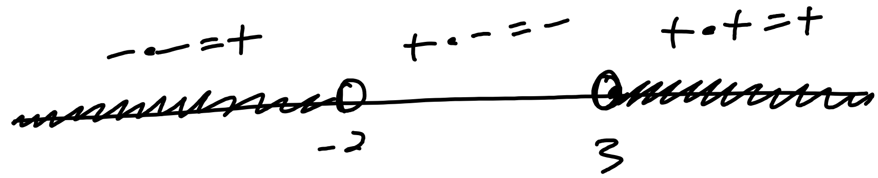


Domain $(-\infty,-2] \cup [3,+\infty)$

example $f(x) = \dfrac{1}{\sqrt{x^2-x-6}}$

Defined as long as $x^2-x-6>0$

$(x-3)(x+2)>0$



Domain $(-\infty,-2)\cup(3,+\infty)$