

1.8 Inequalities with absolute-value expressions.

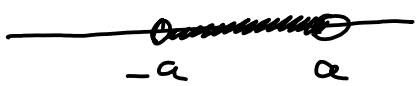
Remember $|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$

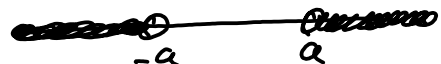
Some properties for absolute values

1. $|a||b| = |ab|$

2. $\frac{|a|}{|b|} = \left| \frac{a}{b} \right|$

3. If $|x| = a$, then $x = a$ or $x = -a$

4. If $|x| < a$, then $-a < x < a$ 

5. If $|x| > a$, then $x < -a$ or $x > a$ 

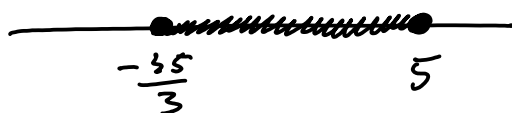
example solve for x

$$|3x + 10| \leq 25$$

$$\begin{array}{cccc} -25 & \leq & 3x + 10 & \leq & 25 \\ -10 & & -10 & & -10 \end{array}$$

$$\frac{1}{3}(-35 \leq 3x \leq 15)$$

$$\boxed{\frac{-35}{3} \leq x \leq 5}$$



$$\left[-\frac{35}{3}, 5 \right]$$

Similar

$$|3x+10| > 25$$

$$3x+10 > 25 \quad \text{or}$$

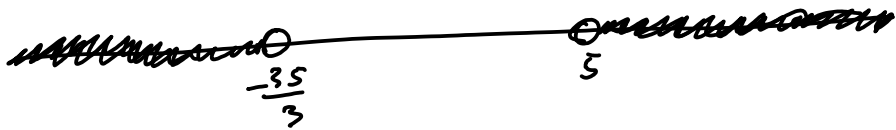
$$3x+10 < -25$$

$$3x > 15$$

$$3x < -15$$

$$x > 5$$

$$x < \frac{-35}{3}$$



example

$$3x + |2x-5| + 1 = 0$$

$$\begin{array}{ccc} -3x & & -1 - 3x \\ & -1 & \end{array}$$

$$|2x-5| = -1-3x$$

This means

$$\begin{array}{ccc} 2x-5 = -(-3x) & \text{or} & 2x-5 = -(-1-3x) \\ +3x & +3x & \end{array}$$

$$2x-5 = 1+3x$$

$$\begin{array}{ccc} 2x-5 = 1+3x \\ -2x & & -2x \end{array}$$

$$5x-5 = -1$$

$$-5 = 1+x$$

$$5x = 4$$

$$\boxed{x = \frac{4}{5}}$$

$$\boxed{-6 = x}$$

similar example

$$3x + |2x - 5| + 1 > 0$$

$-3x \qquad -1 \quad -1-3x$

$$|2x - 5| > -1 - 3x$$

$$2x - 5 > -1 - 3x \quad \text{or} \quad 2x - 5 < -(-1 - 3x)$$

$$5x - 5 > -1$$

$$5x > 4$$

$$x > \frac{4}{5}$$

$$2x - 5 < 1 + 3x$$

$$-5 < 1 + x$$

$$-6 < x$$



solution is just $x > -6$, $(-6, +\infty)$.

example

$$3 - 2|2x+4| \leq 1$$

$-\frac{1}{2} (-2|2x+4| \leq -2)$ make sure to flip inequality

$$|2x+4| \geq 1$$

$$2x+4 \geq 1$$

or

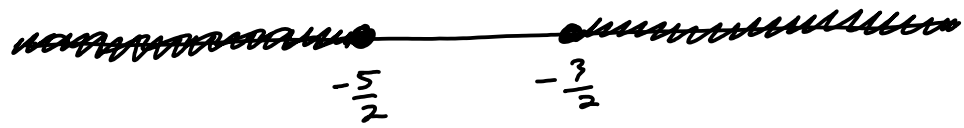
$$2x+4 \leq -1$$

$$2x \geq -3$$

$$x \geq -\frac{3}{2}$$

$$2x \leq -5$$

$$x \leq -\frac{5}{2}$$



$$\left(-\infty, -\frac{5}{2}\right] \cup \left[-\frac{3}{2}, +\infty\right)$$

example

$$\left| \frac{6}{2x-3} \right| < 60$$

undefined when

$$2x-3=0$$

$$x = \frac{3}{2}$$

$$\frac{|6|}{|2x-3|} < 60$$

$$\cancel{2x-3} \frac{6}{\cancel{2x-3}} < 60 \quad |2x-3|$$

$$\frac{1}{60} (6 < 60 |2x-3|)$$

$$\frac{1}{10} < |2x-3|$$

$$2x-3 > \frac{1}{10}$$

or

$$2x-3 < -\frac{1}{10}$$

$$2x > \frac{31}{10}$$

$$2x < \frac{29}{10}$$

$$x > \frac{31}{20}$$

$$x < \frac{29}{20}$$

