Section 1.7 Solving some types of inequalities.

Some basic rules for working with inequalities are as follows

Assume $A \le B$

1. $A \pm C \le B \pm C$ (add or subtract from each side and inequality is preserved)

2. If $k > 0$ Then

$$kA \le kB, \quad \frac{A}{k} \le \frac{B}{k}, \quad -kA \ge -kB, \quad \frac{A}{-k} \ge \frac{B}{-k}$$

3. If $0 < A \le B$, Then $\frac{1}{A} \ge \frac{1}{B}$

Similar rules for $A < B$.

Solving linear inequalities

example Find all x for which

$$3x - 5 < x + 9$$

$$+5 \qquad\qquad +5$$

$$3x < x + 14$$

$$-x \qquad -x$$

$$2x < 14$$

$$\tfrac{1}{2}\,2x < \tfrac{1}{2}\,14$$

$$\boxed{x < 7} \qquad (-\infty, 7)$$

example The rules apply for a 3-way inequality.

$$-4 \le 5 - 3x < 9$$

$$-5 \qquad -5 \qquad\qquad -5$$

$$-9 \le -3x < 4$$

$$\frac{-9}{-3} \ge \frac{-3x}{-3} > \frac{4}{-3}$$

interval notation $\left(-\frac{4}{3}, 3\right]$

$$\boxed{3 \ge x > -\frac{4}{3}}$$

polynomial inequalities

If we can fully factor a polynomial $p(x)$, then we can solve $p(x) > 0$ or $p(x) \geq 0$

Remember

$+ \cdot + = +$ $\qquad + \cdot - \cdot - = +$ $\qquad \dfrac{- \cdot - \cdot +}{- \cdot +} = -$

$+ \cdot - = -$ $\qquad - \cdot - \cdot - = -$

$- \cdot - = +$ $\qquad - \cdot + \cdot + = -$ $\qquad \dfrac{+ \cdot - \cdot +}{- \cdot +} = +$

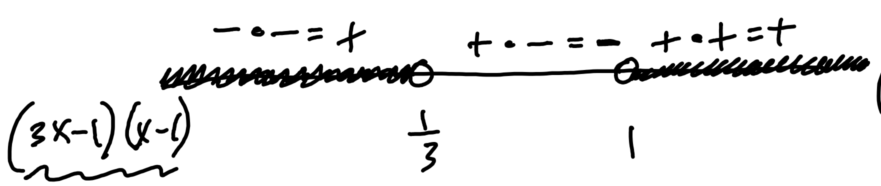
example Find all x for which

$$3x^2 - 4x + 1 > 0$$ ① factor fully

$$(3x-1)(x-1) > 0$$ ② Solve $(3x-1)(x-1) = 0$

$3x - 1 = 0$ or $x - 1 = 0$

$\boxed{x = \frac{1}{3}}$ $\qquad \boxed{x = 1}$



③ graph these solutions a number line with open/closed dots as appropriate.

interval notation $\left(-\infty, \frac{1}{3}\right)$ and $(1, +\infty)$

inequalities $x < \frac{1}{3}$ and $x > 1$

④ test the intervals and shade in the solutions example Find all x for which

$$2x^3+x^2-18x-9 \geq 0$$

$$(2x^3+x^2)+(-18x-9) \geq 0$$

$$x^2(2x+1)-9(2x+1) \geq 0$$

$$(x^2-9)(2x+1) \geq 0$$

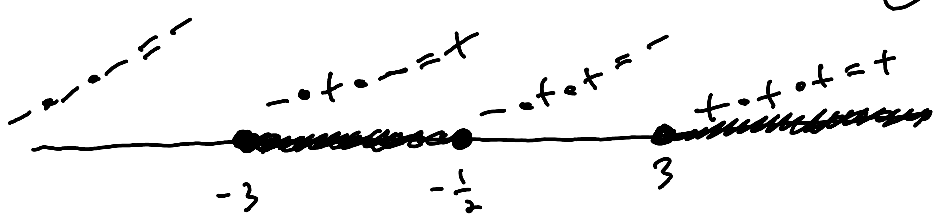
$$(x-3)(x+3)(2x+1) \geq 0$$

① factor fully

② solve equation

$$(x-3)(x+3)(2x+1)=0$$

$x=3$, $x=-3$, $x=-\frac{1}{2}$



$(x-3)(x+3)(2x+1)$

③ Graph solutions on number line

④ Test intervals and state the solution

$$\boxed{\begin{gathered}\left[-3,-\tfrac{1}{2}\right] \text{ and } [3,+\infty) \\[2ex] -3 \leq x \leq -\tfrac{1}{2} \text{ and } 3 \leq x\end{gathered}}$$

Inequalities $\frac{p(x)}{q(x)} > 0$ or $\frac{p(x)}{q(x)} \leq 0$ can be solved with similar techniques. Remember, a fraction $\frac{?}{0}$ is undefined. So if $x=4$ has form $\frac{?}{0}$, then there is an open dot at $x=4$.

<u>example</u> Find all x satisfying

$$\frac{x+1}{(x-1)^2(x+4)} \geq 0$$

① factor fully

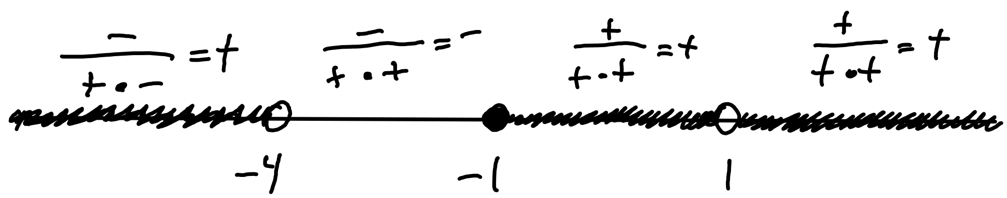
② find solutions to numerator $= 0$ and denominator $= 0$ and plot on a number line.

numerator $= 0$
$x=-1$

denominator $= 0$
$x=-4$, $x=1$

③ test intervals

④ write down solution



$\frac{x+1}{(x-1)^2(x+4)}$

$$(-\infty,-4) \cup [-1,1) \cup (1,+\infty)$$

example

$$\frac{3+x}{3-x} \le 1$$

$$-1 \quad -1$$

$$\frac{3+x}{3-x} - 1 \le 0$$

$$\frac{3+x}{3-x} - \frac{3-x}{3-x} \le 0$$

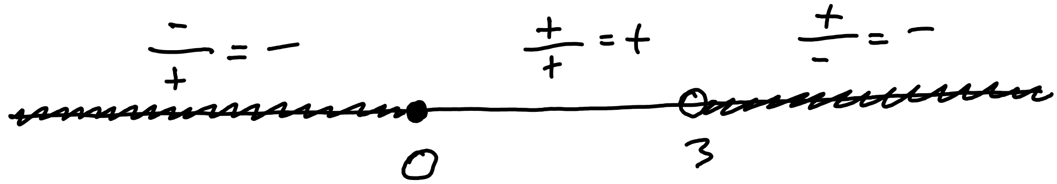
$$\frac{3+x-(3-x)}{3-x} \le 0$$

$$\frac{3+x-3+x}{3-x} \le 0$$

$$\frac{2x}{3-x} \le 0$$

$x=0$ numerator $=0$

$x=3$ denominator $=0$



$$\frac{2x}{3-x}$$

$[-\infty, 0]$ and $(3, +\infty)$

$x \le 0$ and $x > 3$