# Section 1.6 Solving other types of equations

## Polynomial Equations

For example, $x^4 - x^3 - 6x^2$ is a polynomial.

$x^4 - x^3 - 6x^2 = 0$ is a polynomial equation.

If a polynomial equation with 0 on one side can be factored into degree-1 factors, then we can solve.

example

$$x^4 - x^3 - 6x^2 = 0$$

$$x^2(x^2 - x - 6) = 0$$

$$x^2(x-3)(x+2) = 0$$

Remember

$abc = 0$ if and only if

$a = 0$ or $b = 0$ or $c = 0$.

$x^2 = 0$ or $x - 3 = 0$ or $x + 2 = 0$

$\sqrt{x^2} = \sqrt{0}$ OR $\boxed{x = 3}$ OR $\boxed{x = -2}$

$\boxed{x = 0}$ OR

example

$$2x^3+x^2-18x-9=0$$

$$(2x^3+x^2)+(-18x-9)=0$$

$$x^2(2x+1)-9(2x+1)=0$$

$$(2x+1)(x^2-9)=0$$

$$(2x+1)(x-3)(x+3)=0$$

$2x+1=0$ or $x-3=0$ or $x+3=0$

$\boxed{x=-\frac{1}{2}}$ $\boxed{x=3}$ $\boxed{x=-3}$

## Rational Expressions

example $\frac{10}{x}-\frac{12}{x-3}=-4$ Least Common Denominator $x(x-3)$

$$x(x-3)\left[\frac{10}{x}-\frac{12}{x-3}\right]=-4x(x-3)$$

$$\frac{10\cancel{x}(x-3)}{\cancel{x}}-\frac{12x\cancel{(x-3)}}{\cancel{x-3}}=-4x^2+12x$$

$$10(x-3)-12x=-4x^2+12x$$

$$10x-30-12x=-4x^2+12x$$

$+4x^2-12x$ $+4x^2-12x$

$$\frac{4x^2 - 14x - 30}{2} = \frac{0}{2}$$

$$2x^2 - 7x - 15 = 0$$

$$(2x + 3)(x - 5) = 0$$

$$2x + 3 = 0 \quad \text{or} \quad x - 5 = 0$$

$$\boxed{x = \frac{-3}{2}} \qquad \boxed{x = 5}$$

## Equations with Nested Fractions

Example

$$\frac{3 + \frac{1}{x}}{2 - \frac{4}{x}} = x$$

Simplify the Nested fraction(s) into one simple fraction.

$$\frac{\frac{3x}{x} + \frac{1}{x}}{\frac{2x}{x} - \frac{4}{x}} = x$$

$$\frac{\frac{3x+1}{x}}{\frac{2x-4}{x}} = x$$

$$\frac{3x+1}{\cancel{x}} \cdot \frac{\cancel{x}}{2x-4} = x$$

$$\frac{\frac{A}{B}}{\frac{C}{D}} = \frac{A}{B}\frac{D}{C}$$

$$\frac{3x+1}{2x-4} = x$$

LCD $(2x-4)$

$$(2x-4)\frac{3x+1}{2x-4} = x(2x-4)$$

$$3x+1 = 2x^2-4x$$
$$-3x-1 \quad -3x-1$$

$$0 = 2x^2-7x-1$$

Doesn't factor, use quadratic formula.

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$a=2$
$b=-7$
$c=-1$

$$x = \frac{7 \pm \sqrt{49+8}}{4}$$

$$x = \frac{7 \pm \sqrt{57}}{4}$$

$$x = \frac{7+\sqrt{57}}{4} \text{ or } x = \frac{7-\sqrt{57}}{4}$$

## equations with $\sqrt{\ }$ in them

If you have $\sqrt{a} = b$ then you can square both sides to obtain

$$a = b^2$$

and this may help.

## example

$\sqrt{4-6x} - 2x = 0$ isolate $\sqrt{\ }$ on one side by itself, first

$+2x \quad +2x$

$\sqrt{4-6x} = 2x$ square both sides

$$\left(\sqrt{4-6x}\right)^2 = (2x)^2$$

$$4 - 6x = 4x^2$$

$-4+6x \quad -4+6x$

$$\frac{1}{2}\left[0 = 4x^2 + 6x - 4\right]$$

$$0 = 2x^2 + 3x - 2$$

$$0 = (2x-1)(x+2)$$

$$2x - 1 = 0 \quad \text{or} \quad x + 2 = 0$$

$\boxed{x=\frac{1}{2}}$ or ~~$\boxed{x=-2}$~~

In this example you to check your answers against the original equation because squaring both sides might give number that dont work in the original equation

$$\sqrt{4-6x}-2x=0$$

$x=\frac{1}{2}$ $\sqrt{4-6\cdot\frac{1}{2}}-2\cdot\frac{1}{2}=\sqrt{4-3}-1=\sqrt{1}-1=1-1=0$

$x=-2$ $\sqrt{4-6(-2)}-2(-2)=\sqrt{4+12}+4=\sqrt{16}+4=4+4=8$

Equations which reduce to quadratics using a substitution.

example $x^4 - 2x^2 - 8 = 0$

$(x^2)^2 - 2x^2 - 8 = 0$

$(x^2 - 4)(x^2 + 2) = 0$

$(x-2)(x+2)(x^2+2) = 0$

$x - 2 = 0$ or $x + 2 = 0$ $\quad x^2 + 2 = 0$

$x = 2$ $\quad x = -2$ $\quad \cancel{x^2 = -2}$ No solutions.

Think of this as $a^2 - 2a - 8 = 0$

$(a-4)(a+2) = 0$

where $a = x^2$