1.4 Solving Quadratic equations

A *quadratic polynomial* is of the form ax^2+bx+c

A *quadratic equation* is of the form $ax^2+bx+c=0$

3 methods for solving

(1) Factoring If the polynomial can be factored, then the factorization yields the solution(s).

example
$$2x^2-x-1=0$$
$$(2x+1)(x-1)=0$$

(For real numbers $ab=0$ if and only if $a=0$ or $b=0$)

$2x+1=0$ or $x-1=0$

$2x=-1$ $x=1$

$x=\frac{-1}{2}$

2 solutions

check $2(1)^2-(1)-1=2-1-1=0$ ✓

$2\left(\frac{-1}{2}\right)^2-\left(\frac{-1}{2}\right)-1=2\frac{1}{4}+\frac{1}{2}-1=\frac{1}{2}+\frac{1}{2}-1=1-1=0$ ✓

(2) Completing the square. This always works if there is a solution. If there isn't a solution, it will show this.

example $x^2-9=0$ no completing the square is necessary.

$x^2=9$

$x=\pm\sqrt{9}$

$x=\pm 3$

$\boxed{x=3 \text{ or } x=-3}$

example $x^2-4x-1=0$ (1) move the constant to the right.

$x^2-4x=1$ (2) add $\left(\frac{4}{2}\right)^2=2^2=4$ to each side.

$+4 \quad +4$

$x^2-4x+4=5$

$(x-2)^2=5$ (3) square root of both sides

$x-2=\pm\sqrt{5}$

$+2 \quad +2$

$x=2\pm\sqrt{5}$

$\boxed{x=2+\sqrt{5} \text{ or } x=2-\sqrt{5}}$

example $x^2 - x + 3 = 0$

$$\quad -3 \quad -3$$

$$x^2 - x = -3$$

$$+\frac{1}{4} \quad +\frac{1}{4}$$

add $\left(\frac{1}{2}\right)^2 = \frac{1}{4}$ to each side

$$x^2 - x + \frac{1}{4} = -3 + \frac{1}{4}$$

$$x^2 - x + \frac{1}{4} = \frac{-11}{4}$$

$$\left(x - \frac{1}{2}\right)^2 = \frac{-11}{4}$$

negative number, no square root, no solution.

example $4x^2 + 5x - 8 = 0$

when the coefficient of x^2 is not 1, divide both sides by the coefficient of x^2.

$$\frac{1}{4}\left(4x^2 + 5x - 8\right) = \frac{1}{4}(0)$$

$$x^2 + \frac{5}{4}x - 2 = 0$$

$$+2 \quad +2$$

$$x^2 + \frac{5}{4}x = 2$$

add $\left(\frac{\frac{5}{4}}{2}\right)^2 = \left(\frac{5}{8}\right)^2 = \frac{25}{64}$ to both sides.

$$+\frac{25}{64} \quad +\frac{25}{64}$$

$$x^2 + \frac{5}{4}x + \frac{25}{64} = 2 + \frac{25}{64}$$

$$x^2 + \frac{5}{4}x + \frac{25}{64} = \frac{153}{64}$$

$$\left(x+\frac{5}{8}\right)^2 = \frac{153}{64}$$

$$x+\frac{5}{8} = \pm\frac{\sqrt{153}}{8}$$

$$x = -\frac{5}{8} \pm \frac{\sqrt{153}}{8}$$

$$x = \frac{-5\pm\sqrt{153}}{8}$$

$$\boxed{x = \frac{-5+\sqrt{153}}{8} \quad \text{or} \quad x = \frac{-5-\sqrt{153}}{8}}$$

3rd Method Quadratic Formula

The solutions to $ax^2+bx+c=0$

are $x = \dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

example Use the quadratic formula to find the solutions to $2x^2-x-1=0$

$a=2$
$b=-1$
$c=-1$

$$x = \frac{1\pm\sqrt{(-1)^2-4(2)(-1)}}{2\cdot 2}$$

$$x = \frac{1\pm\sqrt{1+8}}{4}$$

$$x = \frac{1 \pm \sqrt{9}}{4}$$

$$x = \frac{1 \pm 3}{4}$$

$$x = \frac{1+3}{4} \quad \text{or} \quad x = \frac{1-3}{4}$$

$$x = 1 \qquad x = \frac{-1}{2}$$

<u>example</u> Use the quadratic formula to find the solutions to $x^2 - x - 1 = 0$

$a = 1$
$b = -1$
$c = -1$

$$\longrightarrow \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{1 \pm \sqrt{1 - 4(1)(-1)}}{2 \cdot 1} = \frac{1 \pm \sqrt{1+4}}{2} = \frac{1 \pm \sqrt{5}}{2}$$

$$\boxed{x = \frac{1+\sqrt{5}}{2} \quad \text{or} \quad x = \frac{1-\sqrt{5}}{2}}$$

This number is called the golden ratio.

(12) Solve by factoring

$$4y^2 - 9y = 28$$
$$-28 \quad -28$$

$4y^2-9y-28=0$

$(4y+7)(y-4)=0$

$7\cdot 4=28$

$4\cdot 4=16$

$16-7=9$

$4y+7=0$ or $y-4=0$

$4y=-7$

$\boxed{y=\frac{-7}{4}}$ $\boxed{y=4}$

(21) Solve by completing the square

$x^2+x-\frac{3}{4}=0$

$x^2+x=\frac{3}{4}$

$\left(\frac{1}{2}\right)^2=\frac{1}{4}$

$+\frac{1}{4}$ $+\frac{1}{4}$

$x^2+x+\frac{1}{4}=1$

$\left(x+\frac{1}{2}\right)^2=1$

$x+\frac{1}{2}=\pm 1$

$x=-\frac{1}{2}\pm 1$

$x=-\frac{1}{2}+1$ or $x=-\frac{1}{2}-1$

$\boxed{x=\frac{1}{2}}$ $\boxed{x=\frac{-3}{2}}$

Complete the square and solve.

(27) $2x^2+7x+4=0$

$\frac{1}{2}\left(2x^2+7x+4\right)=\frac{1}{2}(0)$

$x^2+\frac{7}{2}x+2=0$

$x^2+\frac{7}{2}x=-2$

$\left(\frac{\frac{7}{2}}{2}\right)^2=\left(\frac{7}{4}\right)^2=\frac{49}{16}$

$+\frac{49}{16} \quad +\frac{49}{16}$

$-2=\frac{-32}{16}$

$x^2+\frac{7}{2}x+\frac{49}{16}=-2+\frac{49}{16}$

$x^2+\frac{7}{2}x+\frac{49}{16}=\frac{17}{16}$

$\left(x+\frac{7}{4}\right)^2=\frac{17}{16}$

$x+\frac{7}{4}=\pm\frac{\sqrt{17}}{4}$

$x=\frac{-7}{4}\pm\frac{\sqrt{17}}{4}$

$x=\frac{-7\pm\sqrt{17}}{4}$

$\boxed{x=\frac{-7+\sqrt{17}}{4} \quad \text{or} \quad x=\frac{-7-\sqrt{17}}{4}}$

(67) Find two numbers whose sum is 55 and whose product is 684.

let x and y be these numbers.

We want

$$x + y = 55 \quad \text{and} \quad xy = 684$$

$$\begin{aligned} -x \qquad & -x \end{aligned}$$

$$y = 55 - x$$

$$x(55 - x) = 684$$

$$55x - x^2 = 684$$

$$-55x + x^2 \quad -55x + x^2$$

$$0 = x^2 - 55x + 684$$

$a = 1$
$b = -55$
$c = 684$

$$x = \frac{55 \pm \sqrt{(55)^2 - 4(684)}}{2}$$

$$x = \frac{55 \pm \sqrt{289}}{2}$$

$$x = \frac{55 \pm 17}{2}$$

$X = \frac{55+17}{2}$ or $X = \frac{55-17}{2}$

$X = 36$ or $X = 19$

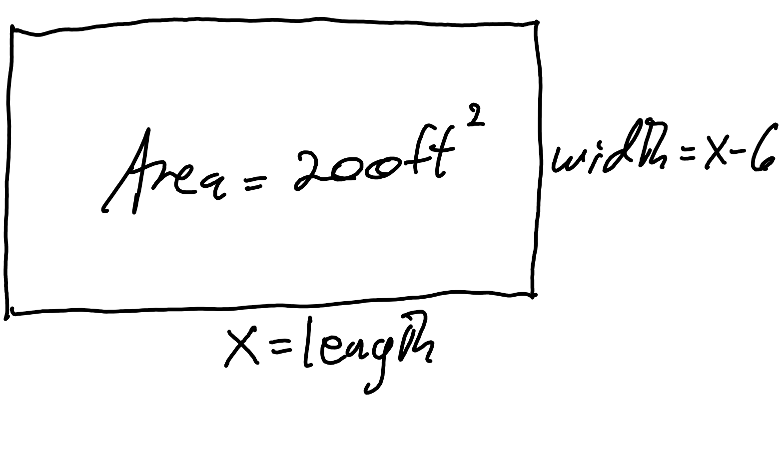
$y = 55 - X$ $\qquad$ $y = 55 - 19$

$y = 19$ $\qquad$ $y = 36$

The two numbers are 19, 36.

like #70

A rectangular room has width 6ft less than its length. If the area of room is 200 ft^2, then what are its dimensions?
(i.e., length, width)



$$\text{length} \cdot \text{width} = \text{area}$$

$$x(x-6) = 200$$

$$x^2 - 6x = 200 \qquad \left(\frac{6}{2}\right)^2 = 3^2 = 9$$

$$+9 \quad +9$$

$$x^2 - 6x + 9 = 209$$

$$(x-3)^2 = 209$$

$$x - 3 = \pm\sqrt{209} \approx \pm 14.45$$

$$x = 3 + 14.45 \quad \text{or} \quad \cancel{x = 3 - 14.45}$$

negative number

$$\boxed{\begin{array}{l} x = 17.45\text{ ft} = \text{length} \\ \text{width} = x - 6 = 11.45\text{ ft} \end{array}}$$

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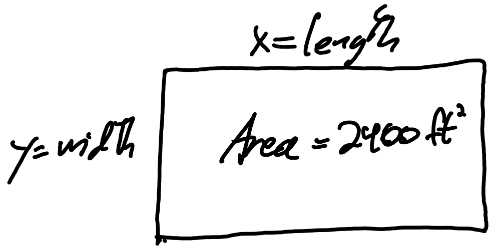

Garden 200ft of fence

Find the length and width.

$$2x + 2y = 200$$

$$x + y = 100$$

$$y = 100 - x$$

$$xy = 2400$$

$$x(100 - x) = 2400$$

$$0 = x^2 - 100x + 2400$$

$0 = (x-60)(x-40)$

$x = 60$ ft or $x = 40$ ft

$y = 100 - x = 40$ $y = 100 - x = 60$

$\boxed{\text{Dimensions of the garden are 60 by 40 ft.}}$