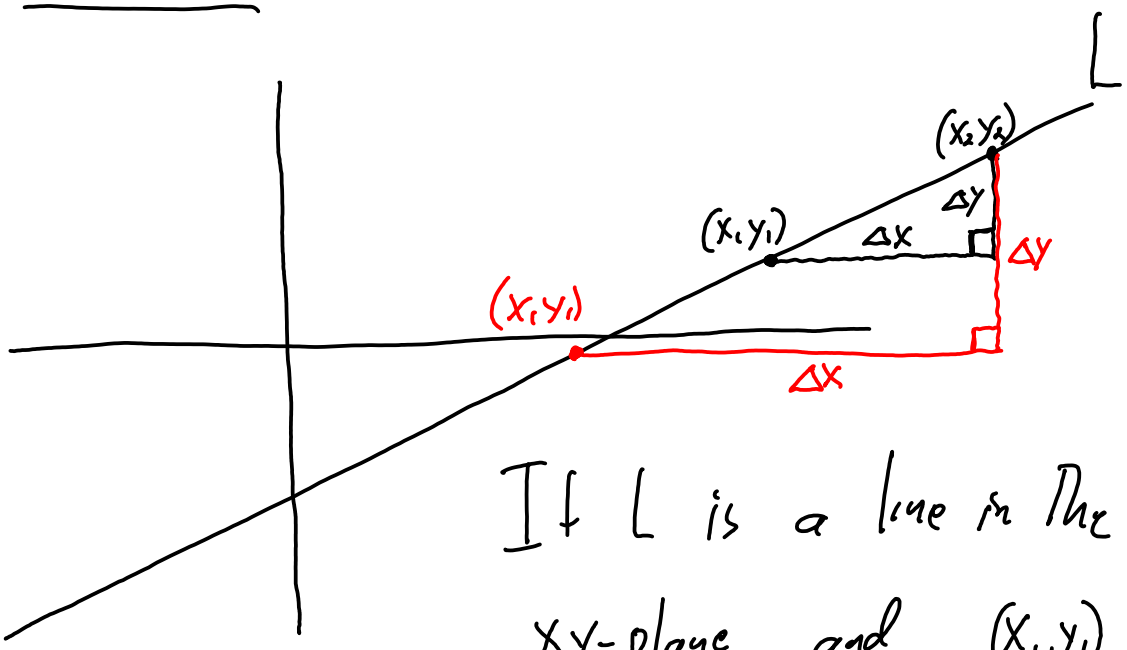
Section 1.3 Lines.

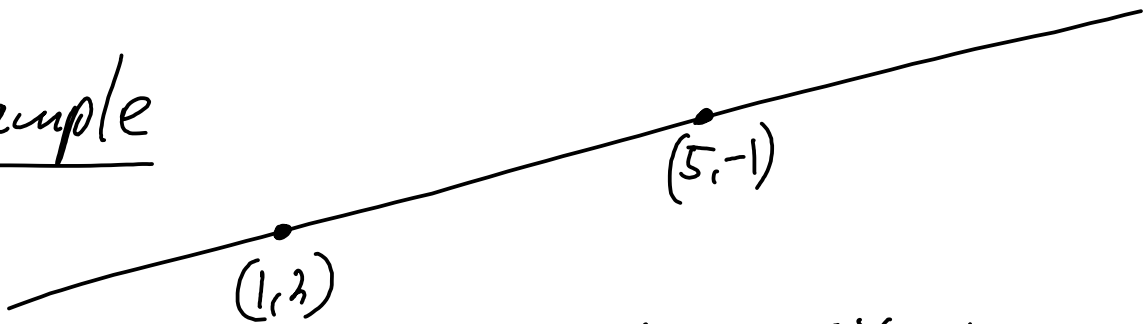


If L is a line in the 2-dimensional xy-plane and (x_1, y_1), (x_2, y_2) are <u>any</u> two points on the line, then

The slope $= \frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{y_1 - y_2}{x_1 - x_2}$

is the same number despite different choices of (x_1, y_1) and (x_2, y_2).
This is from the principle of similar triangles.

example



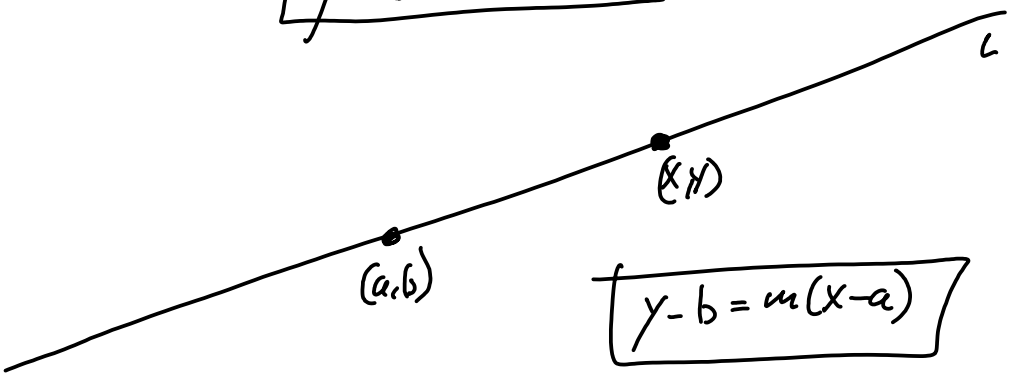
$$\text{Slope} = \frac{\Delta y}{\Delta x} = \frac{-1-2}{5-1} = \frac{-3}{4}$$

So now if (a,b) is any point on Line L and the slope of line L is m, then any other point (x,y) on L must satisfy

$$\frac{y-b}{x-a} = m$$

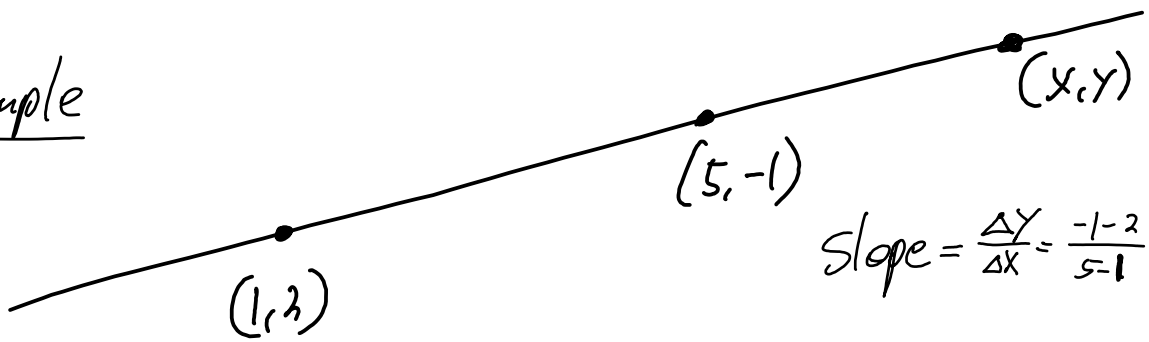
multiplying both sides by $x-a$ yields what is known as the "point-slope" equation of a line

$$\boxed{y-b = m(x-a)}$$



$$\boxed{y-b = m(x-a)}$$

Example



$$\text{Slope} = \frac{\Delta Y}{\Delta X} = \frac{-1-2}{5-1} = \frac{-3}{4}$$

Point-slope equation for this line is

$$y-b = m(x-a)$$

$$\boxed{y-2 = \tfrac{-3}{4}(x-1)} \quad \text{or} \quad \boxed{y+1 = \tfrac{-3}{4}(x-5)}$$

The point-slope equation of a line is <u>not</u> unique to that line because any fixed point (a,b) on the line can be used.

Slope-Intercept Equation

A line L is uniquely represented by its slope-intercept equation

$$y = mx + I$$

m: slope, I: y-intercept.

<u>example</u> From the previous example we know that

$$y-2=\frac{-3}{4}(x-1) \quad \text{or} \quad y+1=\frac{-3}{4}(x-5)$$

These two equations are for the same line. If we solve for y in terms of x then we will get the same slope-intercept equation.

$$y-2=\frac{-3}{4}(x-1)$$
$$y=\frac{-3}{4}(x-1)+2$$
$$y=\frac{-3}{4}x+\frac{3}{4}+2$$
$$\boxed{y=\frac{-3}{4}x+\frac{11}{4}}$$

$$y+1=\frac{-3}{4}(x-5)$$
$$y=\frac{-3}{4}(x-5)-1$$
$$y=\frac{-3}{4}x+\frac{15}{4}-1$$
$$\boxed{y=\frac{-3}{4}x+\frac{11}{4}}$$

Graphing a line from slope-intercept equation

Given a slope-intercept equation $y = mx + I$,

For example $y = \frac{1}{2}x - 3$

How can we get a rough sketch of the line?

① you know the y-intercept $(0, I)$ ($(0, -3)$ in our example)

② Set $y = 0$ to get the x-intercept.

③ Now you have 2 points on the line and you can draw from there.

$y = \frac{1}{2}x - 3$

$(0, -3)$ y-intercept.

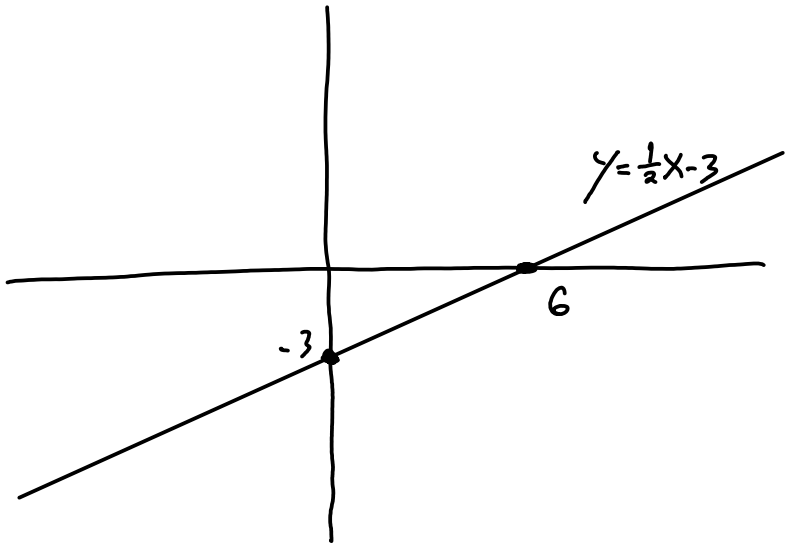
let $y = 0$

$0 = \frac{1}{2}x - 3$

$2 \cdot 3 = \frac{1}{2}x \cdot 2$

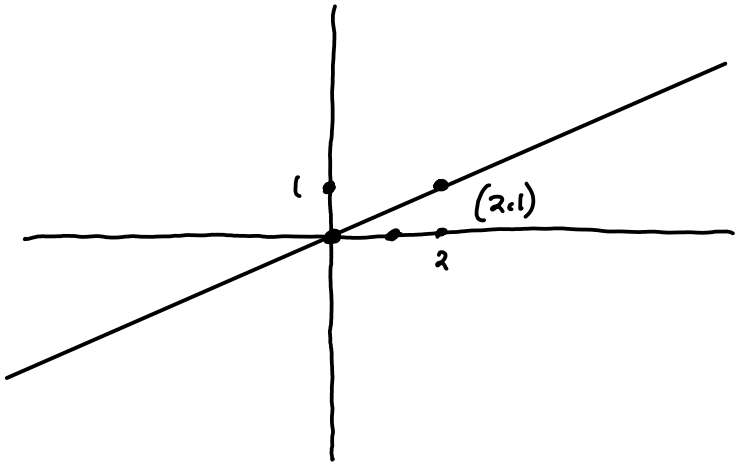
$6 = x$

$(6, 0)$ x-intercept



The only time this technique doesn't work is for lines whose x- and y-intercepts are at the origin

<u>example</u> $y = \frac{1}{2}x$

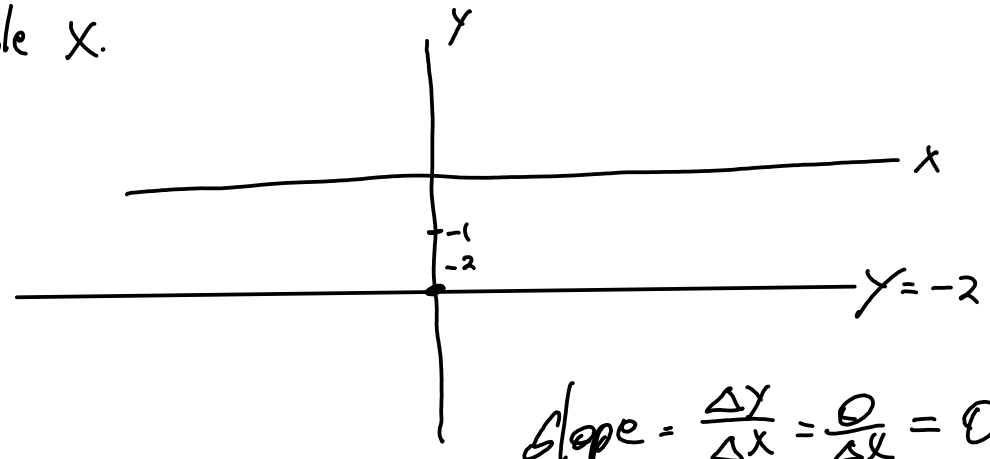


Horizontal and vertical lines

When we refer to the line $y = -2$,

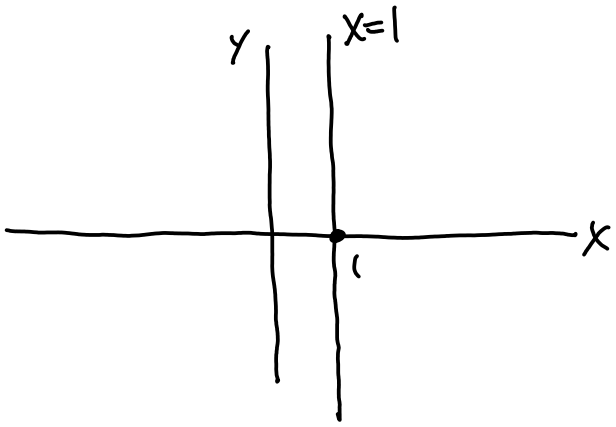
We mean the line whose points are $(x, -2)$

for all possible x.



$$\text{slope} = \frac{\Delta y}{\Delta x} = \frac{0}{\Delta x} = 0$$

When we refer to the line $x=1$ we mean the line whose points are all possible $(1, y)$ for any y.



$$\text{Slope} = \frac{\Delta y}{\Delta x} = \frac{\Delta y}{0}$$

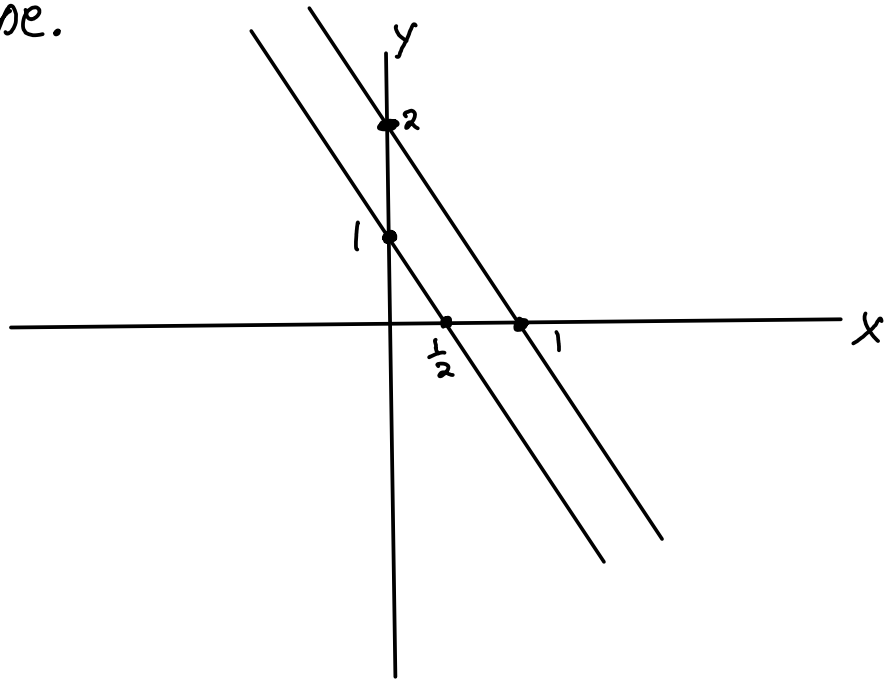
undefined.

Parallel and perpendicular lines

Two lines are parallel when they have the same slope.

$y = -2x + 1$

$y = -2x + 2$

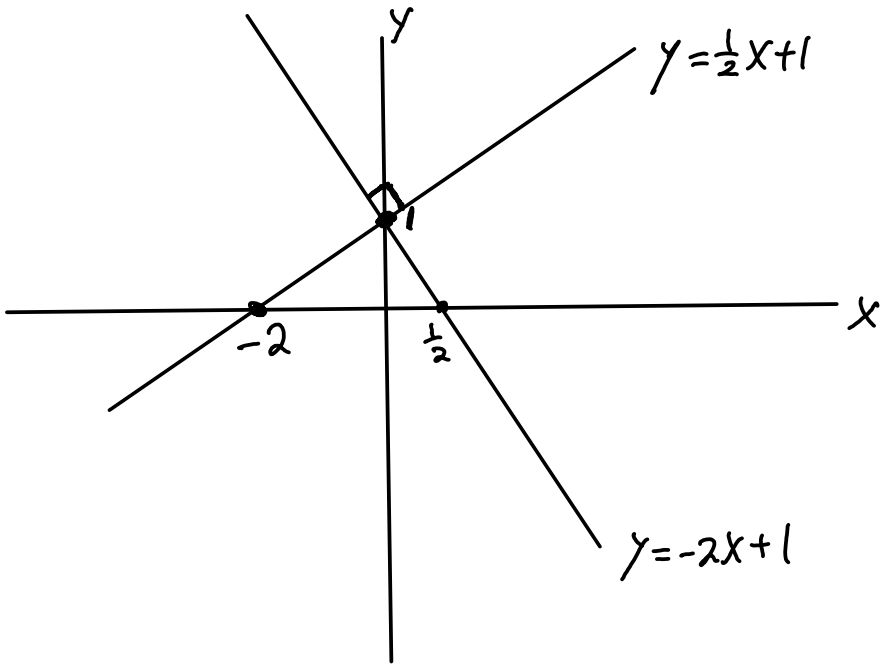


Two lines are <u>perpendicular</u> when the slopes are m and $\frac{-1}{m}$. (Negative reciprocals)

<u>example</u>

$y = -2x + 1$

$y = \frac{1}{2}x + 1$



(24) Find the equation of the line with

slope $= \frac{2}{5}$ y-intercept $= 4$

$\boxed{y = \frac{2}{5}x + 4}$ slope/intercept

(27) slope $= \frac{2}{3}$ contains point $(1, 7)$

$\boxed{y - 7 = \frac{2}{3}(x - 1)}$ point/slope

$y = \frac{2}{3}(x - 1) + 7$

$y = \frac{2}{3}x - \frac{2}{3} + 7$

$\boxed{y = \frac{2}{3} + \frac{19}{3}}$

either answer is acceptable.

(30) line through $(-1,-2)$ and $(4,3)$

$$\text{slope} = \frac{\Delta y}{\Delta x} = \frac{3-(-2)}{4-(-1)} = \frac{5}{5} = 1$$

$$\boxed{y-3 = (x-4)}$$

$$\boxed{y = x-1}$$

(36) line through $(-1,4)$ slope undefined.

vertical line $\boxed{x=-1}$

(39) line through $(1,2)$ and parallel to $y=3x-5$

$$\text{slope} = 3$$

$$y-b = m(x-a)$$

$$\boxed{y-2=3(x-1)}$$

Similar question. line through $(1,2)$ perpendicular to $y=3x-5$

$$\text{slope} = -\frac{1}{3}$$

$$\boxed{y-2=-\frac{1}{3}(x-1)}$$