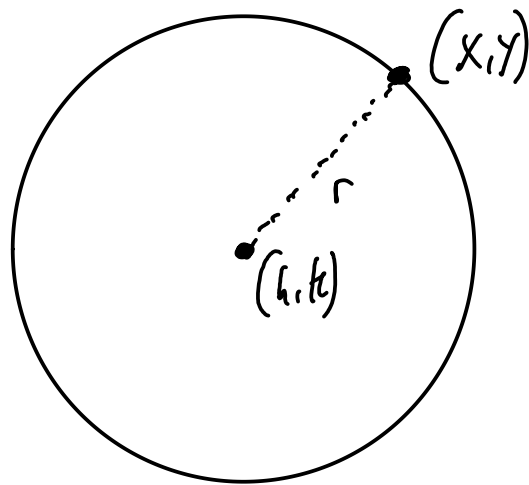


Section 1.2 Circles in the plane.

A circle whose center is (h, k) and whose radius is r is the set of all points (x, y) whose distance from (h, k) is exactly r .



Thus all points (x, y) satisfy

$$r = \sqrt{(x-h)^2 + (y-k)^2}$$

or more usually

$$\boxed{r^2 = (x-h)^2 + (y-k)^2}$$

← equation for
circle with
center = (h, k)
radius = r .

examples ① Circle with center = $(3, -1)$ radius = 2

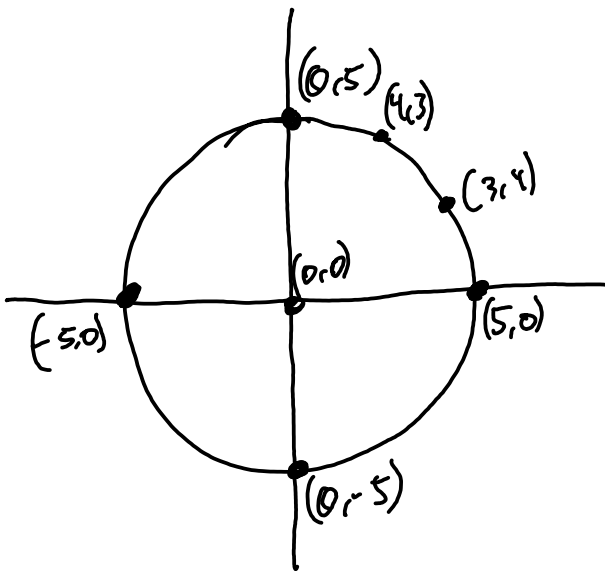
$$(x-3)^2 + (y-(-1))^2 = 2^2$$

$$(x-3)^2 + (y+1)^2 = 4$$

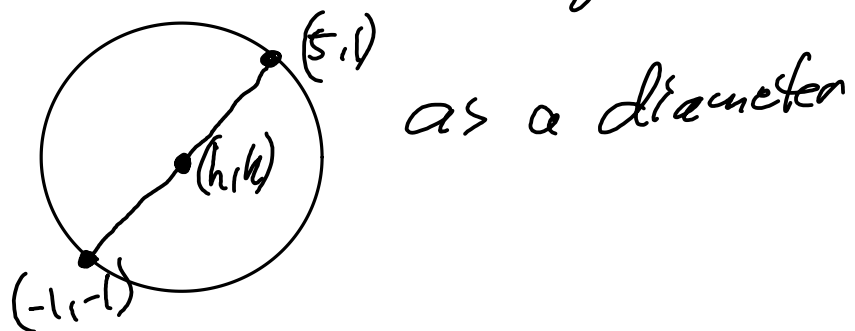
② Circle centered at the origin $(0,0)$ with radius 5 has equation

$$(x-0)^2 + (y-0)^2 = 5^2$$

$$x^2 + y^2 = 25$$



③ Find the circle with segment



midpoint of the diameter is the center

$$(h, k) = \left(\frac{5+(-1)}{2}, \frac{1+(-1)}{2} \right) = \left(\frac{4}{2}, \frac{0}{2} \right) = \underline{(2, 0)}$$

radius = $\frac{1}{2}$ length of a diameter.

$$d = \sqrt{(5-(-1))^2 + (1-(-1))^2} = \sqrt{6^2 + 2^2} = \sqrt{40}$$

So the equation of this circle is

$$(x-2)^2 + (y-0)^2 = (\sqrt{40})^2$$

$$\boxed{(x-2)^2 + y^2 = 40}$$

x- and y-intercepts of a circle

$$\text{Given } (x-h)^2 + (y-k)^2 = r^2$$

The x-intercepts are the solutions to the equation with $y=0$.

The y -intercepts are the solutions to the equation with $x=0$.

example Circle with center = $(3, -1)$ radius = 2 has equation

$$(x-3)^2 + (y+1)^2 = 4$$

Find the x - and y -intercepts.

y -intercepts

$$(0-3)^2 + (y+1)^2 = 4$$

$$9 + (y+1)^2 = 4$$

-9 -9

$$(y+1)^2 = -5$$

No solutions

x -intercepts

$$(x-3)^2 + (0+1)^2 = 4$$

$$(x-3)^2 + 1 = 4$$

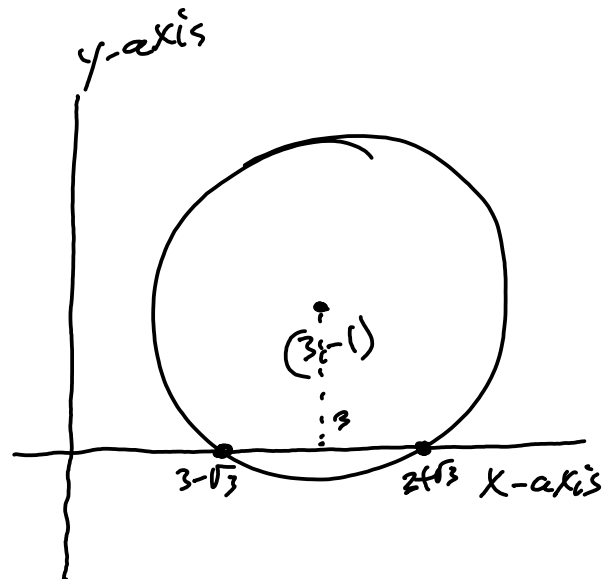
$$(x-3)^2 = 3$$

$$x-3 = \pm\sqrt{3}$$

+3 +3

$$x = 3 \pm \sqrt{3}$$

$$x = 3 + \sqrt{3} \text{ and } x = 3 - \sqrt{3}$$



Completing The square

The algebraic technique of completing The square can take the equation of a circle not in standard form to standard form.

example

$$x^2 - 2x + y^2 - 6y = 0$$

$$\left(\frac{2}{2}\right)^2 = 1^2 = 1 \quad \left(\frac{6}{2}\right)^2 = 3^2 = 9$$

is The equation of a circle. Find its center and radius.

add 1 and add 9 to each side of the equation

$$(x^2 - 2x + 1) + (y^2 - 6y + 9) = 1 + 9$$

$$(x-1) + (y-3)^2 = 10 \quad \text{Standard form.}$$

$$\begin{array}{l} \text{center} = (1, 3) \\ \text{radius} = \sqrt{10} \end{array}$$