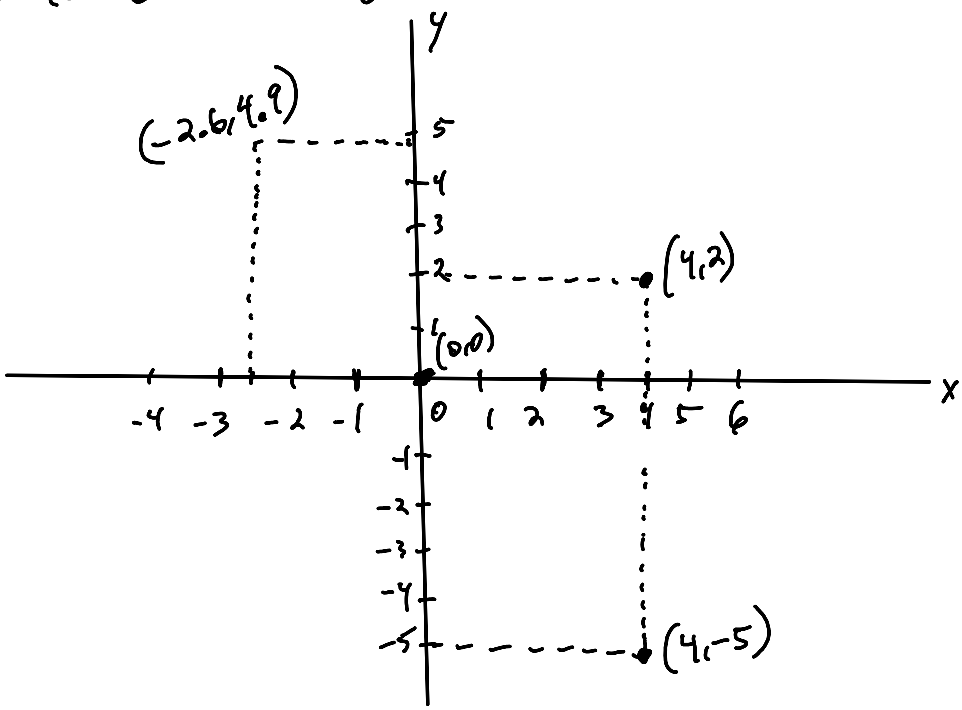
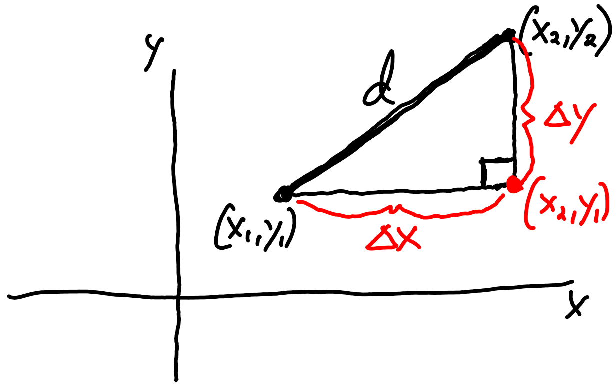
Section 1.1 The 2-dimensional coordinate plane

The 2-dimensional coordinate plane (sometimes called the xy-plane or ty-plane) is defined by two real number lines meeting at a 90° angle at the 0-coordinates of each line.



Each point in the plane is defined by a pair of numbers (x, y).

The straight-line distance between any two points in the xy-plane is given by the Pythagorean Theorem.



$\Delta x = |x_2 - x_1|$

$\Delta y = |y_2 - y_1|$

Pythagorean Theorem

$$d^2 = (\Delta x)^2 + (\Delta y)^2$$

Usually this is written in terms of d rather than d^2

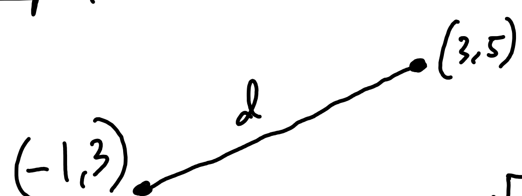
as

$$d = \sqrt{(\Delta x)^2 + (\Delta y)^2}$$

or

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

<u>Example</u>



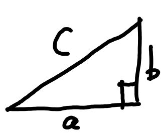
$$d = \sqrt{(\Delta x)^2 + (\Delta y)^2}$$

$\Delta x = |-1-3| = 4$

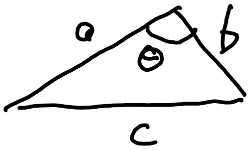
$\Delta y = |5-3| = 2$

$$d = \sqrt{4^2 + 2^2} = \sqrt{20} = \sqrt{4 \cdot 5} = \sqrt{4}\sqrt{5} = 2\sqrt{5}$$

<u>Formal Statement of the Pythagorean Theorem</u>

If  is a right triangle, then $a^2 + b^2 = c^2$.

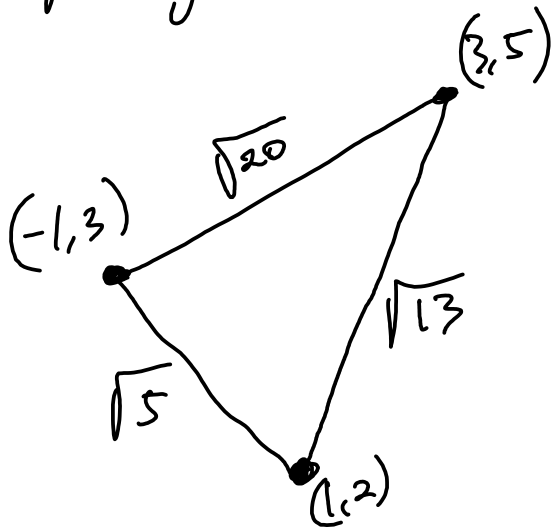
A "converse" statement also holds.

Let 

be a triangle.

If $a^2+b^2=c^2$, then $\theta=90°$ in otherwords the triangle is a right triangle.

<u>example</u> Is the following triangle a right triangle?



$$\sqrt{(3-1)^2+(5-2)^2} =$$
$$\sqrt{2^2+3^2} = \sqrt{13}$$

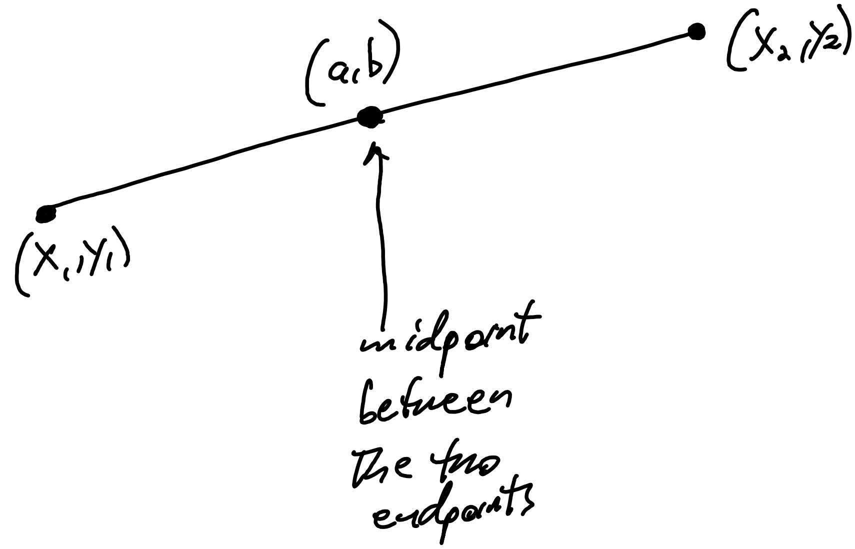
$$\sqrt{(1-(-1))^2+(3-2)^2} =$$
$$\sqrt{2^2+1^2} = \sqrt{5}$$

$$(\sqrt{5})^2+(\sqrt{13})^2 \neq (\sqrt{20})^2$$
$$5+13 \neq 20$$
$$18 \neq 20$$

So this is not a right triangle.

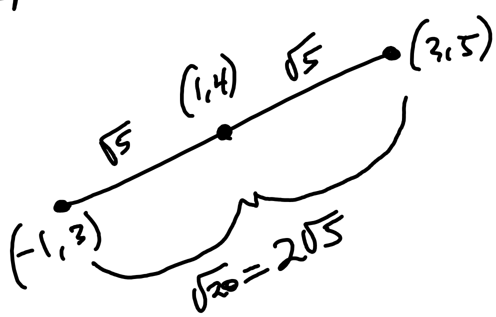
Midpoints

Given a line segment between two points



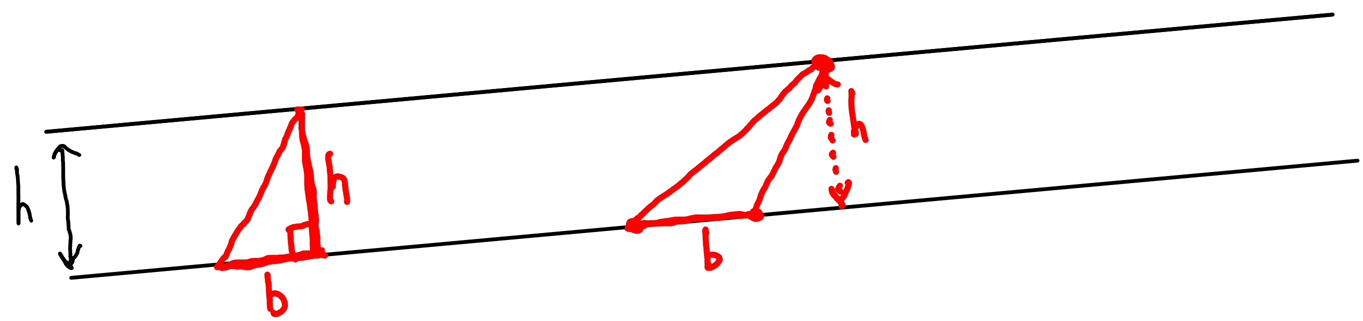
$$(a,b) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

example



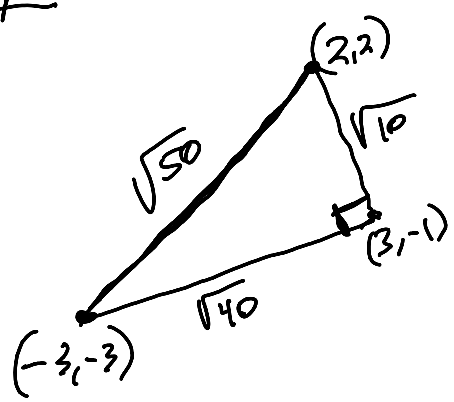
$$(a,b) = \left(\frac{-1+3}{2}, \frac{3+5}{2}\right) = \left(\frac{2}{2}, \frac{8}{2}\right) = (1,4)$$

Areas of triangles



$$\text{area} = \frac{1}{2}bh$$

example



$$\sqrt{(3-(-3))^2+(3-1)^2} = \sqrt{6^2+2^2} = \sqrt{40}$$

$$\sqrt{(3-2)^2+(2-(-1))^2} = \sqrt{1^2+3^2} = \sqrt{10}$$

$$\sqrt{(-3-2)^2+(-3-2)^2} = \sqrt{5^2+5^2} = \sqrt{50}$$

$$\sqrt{50}^2 = \sqrt{40}^2 + \sqrt{10}^2$$

$$50 = 40+10$$

So this triangle is a right triangle.

$$\text{Area} = \frac{1}{2}\sqrt{40}\,\sqrt{10} = \sqrt{\frac{1}{4}}\,\sqrt{400} = \sqrt{100} = 10$$