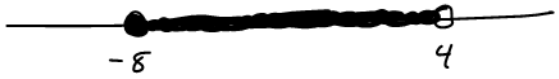
Math1280 – Fall 2023 – Exam 1 **Name**_____

(1) Write the inequalities that describe the interval $[-8, 4)$ and graph the interval on a number line.

$[-8, 4)$ $\quad -8 \leq x < 4$



(2) Simplify $\left(\frac{q^{-1}rs^2}{r^{-3}sq^{-2}}\right)^2$

$$\left(\frac{q^{-1}r s^2}{r^{-3}sq^{-2}}\right)^2 = \frac{q^{-2}r^2s^4}{r^{-6}s^2q^{-4}} = \frac{r^2s^4r^6q^4}{q^2s^2} = \frac{r^8s^4q^4}{s^2q^2} = \boxed{q^2r^8s^2}$$

(3) Perform the following multiplication of polynomials and simplify. $(2x^2 - x - 3)(x + 1)$

$$(2x^2 - x - 3)(x+1) = 2x^3 - x^2 - 3x + 2x^2 - x - 3$$

$$= 2x^3 + x^2 - 4x - 3$$

(4) Subtract the two rational expressions and then simplify. $\dfrac{1}{x} - \dfrac{1}{x^2 + x}$

$$\frac{1}{x} - \frac{1}{x^2+x} = \frac{1}{x} - \frac{1}{x(x+1)} = \frac{1}{x}\cdot\frac{x+1}{x+1} - \frac{1}{x(x+1)} =$$

$$\frac{x+1}{x(x+1)} - \frac{1}{x(x+1)} = \frac{x+1-1}{x(x+1)} = \frac{\cancel{x}}{\cancel{x}(x+1)} = \frac{1}{x+1}$$

(5) Simplify the compound fraction expression to a single fraction. $\dfrac{\frac{1}{2}-\frac{1}{x}}{x-2}$

$$\frac{\frac{1}{2}-\frac{1}{x}}{x-2} = \frac{\frac{1}{2}\frac{x}{x}-\frac{1}{x}\frac{2}{2}}{\frac{x-2}{1}} = \frac{\frac{x-2}{2x}}{\frac{x-2}{1}} = \frac{x-2}{2x}\frac{1}{x-2} = \frac{\cancel{x-2}}{2x\cancel{(x-2)}} = \boxed{\frac{1}{2x}}$$

(6) Solve the equation. $5(x+1) = x+17$

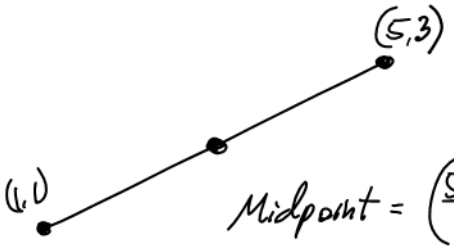
$$5(x+1) = x+17$$

$$5x+5 = x+17$$
$$-x-5 \quad -x-5$$

$$\frac{4x}{4} = \frac{12}{4}$$

$$\boxed{x=3}$$

(7) Find the equation of the circle having the segment from $(1, 1)$ to $(5, 3)$ as a diameter.



Midpoint $= \left(\frac{5+1}{2}, \frac{3+1}{2}\right) = (3, 2)$

$(x-h)^2 + (y-k)^2 = r^2$

length $= \sqrt{4^2 + 2^2}$

$= \sqrt{20}$

radius $= \frac{1}{2}\sqrt{20} = \frac{1}{2} 2\sqrt{5} = \sqrt{5}$

$\boxed{(x-3)^2 + (y-2)^2 = 5}$

(8) Find the slope-intercept form for the equation of the line with slope $-\frac{1}{3}$ and passing through the point $(6, -1)$. Sketch the line.

$y - b = m(x-a)$

$y+1 = \frac{-1}{3}(x-6)$

$y+1 = \frac{-1}{3}x + 2$

$\boxed{y = \frac{-1}{3}x + 1}$

y-intercept

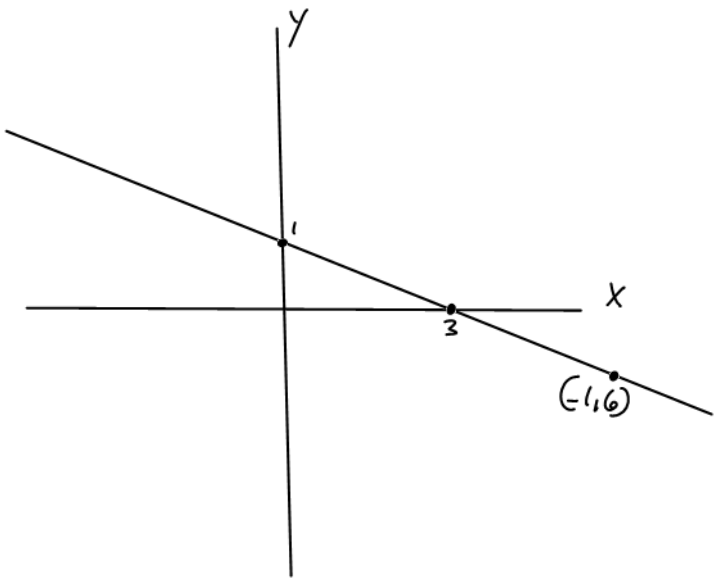
$y = 1$

x-intercept

$0 = \frac{-1}{3}x + 1$

$\frac{1}{3}x = 1$

$x = 3$



(9) Solve this quadratic equation by factoring. $3x^2 + 2x - 1 = 0$.

$3x^2+2x-1=0$

$(3x-1)(x+1)=0$

$3x-1=0$ or $x+1=0$

$\boxed{x=\frac{1}{3}}$ or $\boxed{x=-1}$

(10) Solve this quadratic equation by completing the square $x^2 + 6x + 2 = 0$.

$x^2+6x+2=0$

$x^2+6x=-2$

$x^2+6x+9=-2+9$

$\sqrt{(x+3)^2}=\sqrt{7}$

$|x+3|=\sqrt{7}$

$x+3=\pm\sqrt{7}$

$x=-3\pm\sqrt{7}$

$\boxed{x=-3+\sqrt{7}}$ OR $\boxed{x=-3-\sqrt{7}}$